Topological Defects, Gravity Waves and Proton Decay

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in collaboration with
\( G = SO(10)/\text{Spin}(10) \)

\( H = SU(3)_c \times U(1)_{\text{e.m.}} \)

\( \Pi_2(G/H) \cong \Pi_1(H) \Rightarrow \text{Monopoles} \)

\( \Pi_1(G/H) \cong \Pi_0(H) = \mathbb{Z}_2 \Rightarrow \text{Cosmic Strings (provided } G \to H \text{ breaking uses only tensor representations)} \)

\( \mathbb{Z}_2 \subset \mathbb{Z}_4 \) (center of \( SO(10) \))

Recent work suggests that this \( \mathbb{Z}_2 \) symmetry can yield plausible cold dark matter candidates.

Intermediate scale monopoles and cosmic strings may survive inflation.

Any unified theory with electric charge quantization predicts the existence of topologically stable ('tHooft-Polyakov) magnetic monopoles. Their mass is about an order of magnitude larger than the associated symmetry breaking scale.

Examples:

1. \( \text{SU}(5) \rightarrow \text{SM} \ (3-2-1) \)
   Lightest monopole carries one unit of Dirac magnetic charge even though there exist fractionally charged quarks;

\[
\text{monopole mass} \sim \frac{M_G}{\alpha_G}
\]
$SU(4)_c \times SU(2)_L \times SU(2)_R$ (Pati-Salam)

Electric charge is quantized with the smallest permissible charge being $\pm (e/6)$; Lightest monopole carries two units of Dirac magnetic charge;

SO(10) → 4-2-2 → 3-2-1

Two sets of monopoles: First breaking produces monopoles with a single unit of Dirac charge. Second breaking yields monopoles with two Dirac units.

$E_6$ breaking to the SM can yield ‘lighter’ monopoles carrying three units of Dirac charge.

$E_6 \rightarrow SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \rightarrow$
$SU(3)_c \otimes SU(2)_L \otimes U(1)_{em}$

The discovery of primordial magnetic monopoles would have far-reaching implications for high energy physics & cosmology.
They are produced via the Kibble Mechanism as $G \rightarrow H$:

Center of monopole has $G$ symmetry $\langle \phi \rangle = 0$

Initial no. density $\propto T_{c}^{-3}$. With big bang cosmology such numbers are unacceptable.

$r_{in} = \frac{N_{m}}{N_{\gamma}} \sim 10^{-2}$.

$\Rightarrow$ Monopole Problem

(Need Inflation)
Cosmic Strings from SO(10)

Consider $SO(10)^{M_{GUT}} \rightarrow SU(4) \times SU(2)_L \times SU(2)_R^{M_I \sim 10^{13} \text{GeV}}$

$SU(3) \times SU(2) \times U(1) \times \mathbb{Z}_2$

Cosmic Strings with $G_\mu \sim 10^{-12}$ should be around
Stochastic gravitational wave backgrounds compared with present and future experiments. The grey lines show the background from cosmic strings with the indicated energy scales $G$. The straight black line is the largest allowable background from SMBBH. [arxiv: 1709.02434]
Successful Primordial Inflation should:

- Explain flatness, isotropy;
- Provide origin of $\frac{\delta T}{T}$;
- Offer testable predictions for $n_s, r$ (gravity waves), $dn_s/d \ln k$;
- Recover Hot Big Bang Cosmology;
- Explain the observed baryon asymmetry;
- Offer plausible CDM candidate;
Slow-roll inflation

Inflation is driven by some potential $V(\phi)$:

**Slow-roll parameters:**

$$\epsilon = \frac{m^2_p}{2} \left( \frac{V'}{V} \right)^2, \quad \eta = m^2_p \left( \frac{V''}{V} \right).$$

The spectral index $n_s$ and the tensor to scalar ratio $r$ are given by

$$n_s - 1 \equiv \frac{d \ln \Delta^2_h}{d \ln k}, \quad r \equiv \frac{\Delta^2_h}{\Delta^2_R},$$

where $\Delta^2_h$ and $\Delta^2_R$ are the spectra of primordial gravity waves and curvature perturbation respectively.

Assuming slow-roll approximation (i.e. $(\epsilon, |\eta|) \ll 1$), the spectral index $n_s$ and the tensor to scalar ratio $r$ are given by

$$n_s \simeq 1 - 6\epsilon + 2\eta, \quad r \simeq 16\epsilon.$$
Slow-roll inflation

- The tensor to scalar ratio $r$ can be related to the energy scale of inflation via

$$V(\phi_0)^{1/4} \approx 3.0 \times 10^{16} \ r^{1/4} \ \text{GeV}.$$  

- The amplitude of the curvature perturbation is given by

$$\Delta^2_R = \frac{1}{24\pi^2} \left( \frac{V/m_p^4}{\epsilon} \right)_{\phi=\phi_0} = 2.43 \times 10^{-9} \ \text{(WMAP7 normalization)}.$$  

- The spectrum of the tensor perturbation is given by

$$\Delta^2_h = \frac{2}{3\pi^2} \left( \frac{V}{m_P^4} \right)_{\phi=\phi_0}.$$  

- The number of e-folds after the comoving scale $l_0 = 2\pi/k_0$ has crossed the horizon is given by

$$N_0 = \frac{1}{m_p^2} \int_{\phi_e}^{\phi_0} \left( \frac{V}{V'} \right) \ d\phi.$$  

Inflation ends when $\max[\epsilon(\phi_e), |\eta(\phi_e)|] = 1.$
Inflation with a Higgs Potential [Kallosh and Linde, 07; Rehman, Shafi and Wickman, 08]

- Consider the following Higgs Potential:
  \[ V(\phi) = V_0 \left[ 1 - \left( \frac{\phi}{M} \right)^2 \right]^2 \]  
  \( \leftarrow \) (tree level)

Here \( \phi \) is a gauge singlet field.

- WMAP/Planck data favors BV inflation \( (r \lesssim 0.1) \).

Note: This is for minimal coupling to gravity
$n_s$ vs. $r$ for Higgs potential, superimposed on Planck and Planck+BKP 68% and 95% CL regions taken from arXiv:1502.01589. The dashed portions are for $\phi > v$. $N$ is taken as 50 (left curves) and 60 (right curves).
$n_s$ vs. $r$ for Coleman–Weinberg potential, superimposed on Planck and Planck+BKP 68% and 95% CL regions taken from arXiv:1502.01589. The dashed portions are for $\phi > v$. $N$ is taken as 50 (left curves) and 60 (right curves).
Let’s consider how much dilution of the monopoles is necessary. $M_I \sim 10^{13}$ GeV corresponds to monopole masses of order $M_M \sim 10^{14}$ GeV. For these intermediate mass monopoles the MACRO experiment has put an upper bound on the flux of $2.8 \times 10^{-16}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$. For monopole mass $\sim 10^{14}$ GeV, this bound corresponds to a monopole number per comoving volume of $Y_M \equiv n_M / s \lesssim 10^{-27}$. There is also a stronger but indirect bound on the flux of $(M_M / 10^{17}$ GeV)$10^{-16}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$ obtained by considering the evolution of the seed Galactic magnetic field.

At production, the monopole number density $n_M$ is of order $H_{x}^3$, which gets diluted to $H_{x}^3 e^{-3N_x}$, where $N_x$ is the number of $e$-folds after $\phi = \phi_x$. Using

$$Y_M \sim \frac{H_{x}^3 e^{-3N_x}}{s},$$

where $s = (2\pi^2 gS/45) T_r^3$, we find that sufficient dilution requires $N_x \gtrsim \ln(H_x/T_r) + 20$. Thus, for $T_r \sim 10^9$ GeV, $N_x \gtrsim 30$ yields a monopole flux close to the observable level.
Relativistic Monopoles at IceCube

b–τ Yukawa coupling unification

Without Supersymmetry
Dominant contributions to the bottom quark mass from the gluino and chargino loop

\[ \delta y_b \approx \frac{g_3^2}{12\pi^2} \frac{\mu m_{\tilde{g}} \tan \beta}{m_1^2} + \frac{y_t^2}{32\pi^2} \frac{\mu A_t \tan \beta}{m_2^2} + \ldots \]

where \( m_1 \approx (m_{\tilde{b}_1} + m_{\tilde{b}_2})/2 \) and \( m_2 \approx (m_{\tilde{t}_2} + \mu)/2 \)

where \( \lambda_b = y_b \) and \( \lambda_t = y_t \)

$b - \tau$ YU in SU(5)

\begin{align*}
0 \leq & \quad m_{10} \quad \leq 20 \text{ TeV} \\
0 \leq & \quad m_{5} \quad \leq 20 \text{ TeV} \\
0 \leq & \quad M_{1/2} \quad \leq 5 \text{ TeV} \\
-3 \leq & \quad A_t / m_{10} \quad \leq 3 \\
-20 \leq & \quad A_{b\tau} / m_{5} \quad \leq 20 \\
1.2 \leq & \quad \tan \beta \quad \leq 60 \\
0 \leq & \quad m_{H_d} \quad \leq 20 \text{ TeV} \\
0 \leq & \quad m_{H_u} \quad \leq 20 \text{ TeV}
\end{align*}

\begin{align*}
R \equiv & \quad \frac{\text{Max}(y_b, y_{\tau})}{\text{Min}(y_b, y_{\tau})} \leq 1.1 \quad b - \tau \text{ YU Condition}
\end{align*}

\begin{align*}
y_b : y_{\tau} = (1 - C) : (1 + 3C), \quad |C| \leq 0.2 \quad b - \tau \text{ QYU Condition}
\end{align*}
<table>
<thead>
<tr>
<th></th>
<th>Point 1</th>
<th>Point 2</th>
<th>Point 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{10}$</td>
<td>2325</td>
<td>5805</td>
<td>3299</td>
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<tr>
<td>$M_5$</td>
<td>4334</td>
<td>5756</td>
<td>4813</td>
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<td>$M_{1/2}$</td>
<td>1317</td>
<td>2478</td>
<td>1002</td>
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<td>$m_{H_d}$</td>
<td>1574</td>
<td>6740</td>
<td>1592</td>
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<td>$m_{H_u}$</td>
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<td>$A_t/m_{10}$</td>
<td>-1.73</td>
<td>-1.65</td>
<td>-1.46</td>
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<td>$\mu$</td>
<td>107.8</td>
<td>714.9</td>
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<td>117</td>
<td>163</td>
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<td>950</td>
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<td>$m_{\chi_{1,2}^0}$</td>
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<td>701.3, 716.4</td>
<td>441.7, 783.3</td>
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<td>1128, 2110</td>
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<td>732.5, 2088</td>
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<td>5361</td>
<td>2369</td>
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<td>7354, 7302</td>
<td>3780, 3839</td>
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<td>2797, 5473</td>
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<td>6007</td>
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<td>5464, 5851</td>
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<td>$\sigma_{SI}(pb)$</td>
<td>0.10 $\times 10^{-8}$</td>
<td>0.72 $\times 10^{-9}$</td>
<td>0.20 $\times 10^{-8}$</td>
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<td>$\sigma_{SD}(pb)$</td>
<td>0.82 $\times 10^{-4}$</td>
<td>0.15 $\times 10^{-5}$</td>
<td>0.59 $\times 10^{-6}$</td>
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<td>$\Omega_{CPM} h^2$</td>
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<td>0.097</td>
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<td>$y_{h,\tau}$</td>
<td>0.50, 0.13, 0.17</td>
<td>0.51, 0.07, 0.1</td>
<td>0.52, 0.16, 0.21</td>
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<tr>
<td>$C$</td>
<td>0.08</td>
<td>0.08</td>
<td>0.07</td>
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</table>
Non-minimal quartic inflation

\[ W = S[\kappa(X\bar{X} - M^2)] + \sigma H_u H_d S \]

\[ \mathcal{L} \supset -\frac{1}{2} M_P^2 (1 + \xi \varphi^2) \mathcal{R} + \frac{1}{2} g^{\mu\nu} (\partial_\mu \varphi)(\partial_\nu \varphi) - \frac{\kappa^2}{16} \varphi^4 \]
### $N_0 = 50$

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<tr>
<th>$\xi$</th>
<th>$\varphi_0/M_P$</th>
<th>$\varphi_e/M_P$</th>
<th>$n_s$</th>
<th>$r$</th>
<th>$\alpha(10^{-4})$</th>
<th>$\kappa$</th>
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<td>0</td>
<td>20.2</td>
<td>2.83</td>
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<td>$9.68 \times 10^{-7}$</td>
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<td>0.00527</td>
<td>20.0</td>
<td>2.77</td>
<td>0.955</td>
<td>0.1</td>
<td>-9.74</td>
<td>$1.73 \times 10^{-6}$</td>
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<tr>
<td>0.119</td>
<td>15.8</td>
<td>2.07</td>
<td>0.961</td>
<td>0.01</td>
<td>-7.70</td>
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<td>1</td>
<td>7.82</td>
<td>1.00</td>
<td>0.961</td>
<td>0.00489</td>
<td>-7.51</td>
<td>$5.01 \times 10^{-5}$</td>
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<tr>
<td>10</td>
<td>2.65</td>
<td>0.337</td>
<td>0.962</td>
<td>0.00426</td>
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<td>$4.67 \times 10^{-4}$</td>
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<tr>
<td>1000</td>
<td>0.267</td>
<td>0.0340</td>
<td>0.962</td>
<td>0.00419</td>
<td>-7.48</td>
<td>$4.63 \times 10^{-2}$</td>
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### $N_0 = 60$

<table>
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<tr>
<th>$\xi$</th>
<th>$\varphi_0/M_P$</th>
<th>$\varphi_e/M_P$</th>
<th>$n_s$</th>
<th>$r$</th>
<th>$\alpha(10^{-4})$</th>
<th>$\kappa$</th>
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<td>0</td>
<td>22.1</td>
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<td>0.1</td>
<td>-7.03</td>
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<tr>
<td>0.0690</td>
<td>18.9</td>
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<td>0.967</td>
<td>0.01</td>
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<tr>
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<td>1.00</td>
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<td>0.968</td>
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<td>$3.92 \times 10^{-4}$</td>
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<tr>
<td>1000</td>
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<td>0.968</td>
<td>0.00296</td>
<td>-5.23</td>
<td>$3.88 \times 10^{-2}$</td>
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</table>
\[ \mu = \sigma \langle S \rangle = \frac{\sigma}{\kappa} m_{3/2} \Rightarrow \sigma = \kappa \frac{\mu}{m_{3/2}}, \quad m_{3/2} \ll M \]

Inflaton Decay (mass \( \approx \kappa M \)):

\[
\Gamma = \frac{\sigma^2}{8\pi} m_\phi = \frac{\sqrt{2}}{8\pi} \kappa^3 \left( \frac{\mu}{m_{3/2}} \right)^2 M
\]

\[
T_r \approx 1.712 \times 10^8 \times \kappa^{3/2} \left( \frac{\mu}{m_{3/2}} \right) M^{1/2} \quad [\text{GeV}]
\]
Phenomenological Constraints:
Gravitino ≠ LSP, \( T_r \leq 10^6 \text{GeV} \) to avoid the gravitino problem

\[
\kappa \leq 0.0324 \left( \frac{m_{3/2}^3}{\mu} \right)^{2/3} M^{-1/3} = 0.0324 M^{-1/3};
\]

\( m_{\tilde{\phi}} > m_{H_u,d} \Rightarrow \sqrt{2} \kappa M \geq 1000 \text{GeV} \)
Red \( T_R = 10^6 \text{GeV} \)
Blue \( m_{\tilde{\phi}} = m_{H_u,d} = 1 \text{ TeV} \)
Black \( r = 0.1 \)

Green shaded region satisfies all conditions,

\[ 3.2 \times 10^{16} \lesssim M(\text{GeV}) \lesssim 2 \times 10^{13} \]
Gravitino LSP:

- Relic density from thermal productions

\[ \Omega h^2 \approx 1.8 \times 10^{-8} \text{GeV} \left( \frac{T_r}{m_{3/2}} \right) \left( \frac{M_3}{3 \text{TeV}} \right)^2 \approx 0.1 \]

\[ \Rightarrow \kappa \approx 0.102 \left( \frac{m_{3/2}^2}{\mu \sqrt{M}} \right)^{2/3} \left( \frac{3 \text{TeV}}{M_3} \right)^{4/3} \]

- BBN constraint on NLSP lifetime:

\[ \tau_{NLSP} \approx 10^4 \text{sec} \left( \frac{100 \text{GeV}}{m_{NLSP}} \right)^5 \left( \frac{m_{3/2}}{1 \text{GeV}} \right)^2 < 1 \text{s} \]
Gravitino LSP:

- Take $\mu \sim 1 TeV \sim m_{NLSP}$
- $\tau_{NLSP} < 1 s \Rightarrow m_{3/2} \leq 3 GeV$
- For various $m_{3/2} = 3, 1, 0.32 GeV$ (from top to bottom)

To be able to find a solution, $m_{3/2} \geq 0.32 GeV$
Hyper-Kamiokande Physics Goals

Solar neutrinos  | CP violation  | Astrophysical neutrinos
In presence of dimension five operators the unification condition is modified to

$$(1+\epsilon_1)^{-1}\alpha_1^{-1}(M_{GUT}) = (1+\epsilon_2)^{-1}\alpha_2^{-1}(M_{GUT}) = (1+\epsilon_3)^{-1}\alpha_3^{-1}(M_{GUT})$$

where,

$$\epsilon_1 : \epsilon_2 : \epsilon_3 :: 1 : 6 : -4$$

$\epsilon_1 = -0.01$, $M_G = 1.8 \times 10^{16}$ GeV and $\Lambda = 5 \times 10^{17}$
Summary

- Unification of all forces remains a compelling idea.
- Grand unification explains charge quantization, predicts monopoles and proton decay.
- Stable cosmic strings predicted in realistic GUT.
- Also explains tiny neutrino masses via seesaw mechanism.
- Non-SUSY gauge coupling unification require new particles/new physics below $M_{GUT}$.
- In non-SUSY inflation with Higgs potential, $r \gtrsim 0.02$ (minimal coupling to gravity).
- SUSY and Non-SUSY models offer plausible dark matter candidates such as TeV mass higgsino, axions....
- Higher dimensional operators can have significant impact on $M_{GUT}$ and proton decay predictions in GUTs; especially if the ultraviolet cut-off scale is closer to $M_{GUT}$
- Find dark matter, gravity waves and proton decay.
Thank You!