Measurement Based Quantum Computing, Graph States, and Near-term Realizations

Miami 2018

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17 December 2018

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1. Quantum Entanglement: Communication and Computation

2. Generating a Graph State

3. Entanglement with Quantum Emitters: Physical Systems
Quantum Entanglement: Communication and Computation
Quantum Technologies

Delft Campus (1.3km)$^1$

- Quantum Key Distribution$^2$
- Distributed Quantum Computing
- Quantum Communications, “Quantum Internet”

- All depend on getting entanglement distributed at great distances
- Natural to do so with photons

Entanglement Distribution

- Create a Bell state $|00\rangle + |11\rangle$, and distribute each part.
- What if the distance is too long? Repeaters.
- No cloning theorem: we cannot just copy the state.

- Light-light coupling is very challenging.
- We can do "entanglement swapping".
- Create entangled pairs, and distribute halves to intermediate sites.¹

Bell Measurement

- Create two Bell states $|00\rangle + |11\rangle$ (remotely), and send one of each pair to an intermediate node.

\[
[|00\rangle + |11\rangle] [|00\rangle + |11\rangle] = |0000\rangle + |1100\rangle + |0011\rangle + |1111\rangle
\]

- Call the middle two qubits the ones that were sent to the intermediate site. The Bell state basis

\[
|\Psi^{\pm}\rangle = |01\rangle \pm |10\rangle \quad \text{and} \quad |\Phi^{\pm}\rangle = |00\rangle \pm |11\rangle
\]
We decomposed the qubits at the intermediate site into the Bell basis.

If we measure in that basis, the entanglement is transferred.

“Qubit fusion” joint measurement only succeeds with probability $1/2^1$

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Entanglement Distribution

- Repeater graph states lets you try repeatedly \(^1\)
- Interior qubits let your “route” entanglement

\(^1\) K. Azuma et al., Nat. Commun. 6, 6787 (2015)
Kinds of Entangled States

\[
|\text{GHZ}_3\rangle = \frac{1}{\sqrt{2}} \left[ |\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle \right]
\]

\[
|W_3\rangle = \frac{1}{\sqrt{3}} \left[ |\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\rangle \right]
\]

- Arbitrary local unitaries cannot convert these two states
- Number of classes increase with number of qubits
- One kind of entanglement for 2 qubits. Two kinds for 3 qubits. 9 kinds for 4 qubits…
- Hard to write down
Graph States

- Vertices represent qubits, edges “entanglement”

- Stabilizers:
  \[ K_G^{(a)} = X^{(a)} \prod_{b \text{ adjacent to } a} Z^{(b)} \]

- Alternatively, initialize each qubit to \(|+\rangle = |0\rangle + |1\rangle\), and run a “controlled-Z” gate between each pair of qubits connected by an edge

- \[ cz = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \]

- This is called a (two dimensional) “cluster state.”

- Universal resource for quantum computation

- Only local measurements required!
Generating a Graph State
Coherent quantum mechanical system that is able to (on demand) produce a photon completely entangled with the emitter state:

\[ |\Psi\rangle = |\psi\rangle |0\rangle_e + |\bar{\psi}\rangle |1\rangle_e \rightarrow |\psi\rangle |0\rangle_e |0\rangle_p + |\bar{\psi}\rangle |1\rangle_e |1\rangle_p \]

Local unitary operations and measurements on the emitter.

CZ operations between emitters

“Ancilla” does not have to emit.
The Bell Pair Graph State

- A “Graph State” is a description of quantum mechanical state
- Let’s first show (something equivalent to) the entangled Bell pair!
- At right we have two dots, each representing a qubit
- By convention, when the nodes of the graph are not connected, they are just a product state of
  \[ |+\rangle = |0\rangle + |1\rangle \]
The Bell Pair Graph State

- Acting on $|+\rangle |+\rangle$, $\text{cz}$ gives

$$\text{cz} \left[ |0\rangle |0\rangle + |0\rangle |1\rangle + |1\rangle |0\rangle + |1\rangle |1\rangle \right]$$

$$= |0\rangle |0\rangle + |0\rangle |1\rangle + |1\rangle |0\rangle - |1\rangle |1\rangle$$

$$= |0\rangle \left[ |0\rangle + |1\rangle \right] + |1\rangle \left[ |0\rangle - |1\rangle \right]$$

$$= |0\rangle |+\rangle + |1\rangle |-\rangle$$

- (Up to a local change of basis)
  that’s the Bell state!

- By convention, a line between qubits is a $\text{cz}$ gate.

- $\text{cz} : |0\rangle |\psi\rangle \rightarrow |0\rangle |\psi\rangle$ and
  $|1\rangle |\pm x\rangle \rightarrow |1\rangle |\mp x\rangle$
Multiple Photons from Same Emitter

- If we “pump” the emitter multiple times, we get larger GHZ states

\[ |0\rangle_e |000\ldots\rangle_p + |1\rangle_e |111\ldots\rangle_p \]

- (We’ll draw the emitter as yellow here)

- Can we make more complicated graphs?
Protocol to produce a one-dimensional cluster state.\(^1\)

We’ll (obviously) need some other tool. That is a single qubit operation, the “Hadamard”

\[
H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
\]

Physically, it is a rotation about an axis 45° between \(x\) and \(z\)

\(H |\pm z\rangle = |\pm x\rangle\)

After a single pump protocol, we get

\[ |\psi\rangle = |0\rangle|0\rangle + |1\rangle|1\rangle \]

Basis changes on photons are easy

\[ |0\rangle|+\rangle + |1\rangle|\rangle \]

We get (exactly) the graph state

We don’t “actually” have to do this: we can just keep track of what local operation was going to be performed on the photon, and correct for that when we finally measure it.
Lindner-Rudolph 1D Cluster State

\[ |0\rangle|+\rangle + |1\rangle|\rangle \]

- Pump the emitter again

\[ |0\rangle|+\rangle|0\rangle + |1\rangle|\rangle|1\rangle \]

- Hit the new photon with \( H \)

\[ |0\rangle|+\rangle|+\rangle + |1\rangle|\rangle|\rangle \]

- Notice that

\[ |+\rangle|+\rangle|+\rangle = |0\rangle|+\rangle|+\rangle + |1\rangle|+\rangle|+\rangle \]

- (Our ket is the graph on the right!)
Lindner-Rudolph 1D Cluster State

\[ |0 \, ++ \rangle + |1 \, -- \rangle \]

- Hit both the emitter and the last photon with $H$

\[ |+ \rangle \, ++ \, |0 \rangle + |- \rangle \, -- \, |1 \rangle \]

- By analogy, it’s clear that we have the graph on the right!
Lindner-Rudolph 1D Cluster State

- Repeat!
Lindner-Rudolph 1D Cluster State

- Repeat!
- Pump
• Repeat!
• Pump
• Hadamards
Universal Computation

- This can be used to perform unitary operations
- Suppose the first qubit is in a state $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$, and then entangled with one qubit of a Bell pair:

$$|\Psi\rangle = \alpha |0\rangle [|0\rangle |+\rangle + |1\rangle |-\rangle] +$$

$$\beta |1\rangle [|0\rangle |+\rangle - |1\rangle |-\rangle]$$

- Measure in $X$ basis

$$|\Psi\rangle = (\alpha + \beta) |0\rangle |+\rangle + (\alpha - \beta) |1\rangle |-\rangle$$

- Sacrifice a qubit, change the state
Universal Computation

- To perform a general unitary, you need to use 5 qubit measurements
- Multiple 1D chains allow for multiple qubits to be processed (in parallel)
- If these are connected, you can also perform entangling gates
- That’s the 2D cluster
• Include the ancilla (black).
  Initialize it to $|+\rangle$. This is just a tensor product with that state, then.

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\[1\text{D. Buterakos et al., Phys. Rev. X 7, 041023 (2017)}\]
Using the Ancilla

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- Entangle it with the CZ gate (see where this is going?)

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- Hadamards

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Detaching a qubit?

- If we measure a qubit in the $z$ basis, we remove the edges attached to it.
- It’s not really hard to see, but you have to stare at it for a while

\[ |+\rangle|0\rangle|+\rangle + |−\rangle|1\rangle|−\rangle \]

Measure the middle bit in the $z$ basis:

either \[ |+\rangle|0\rangle|+\rangle \text{ or } |−\rangle|1\rangle|−\rangle \]

Need to know the measurement result to perform some local rotations!
Detaching a qubit?

- Easy to measure that last emitted photon!
Make another leg!

- Re-initializing the emitter and using the CZ gate, we can get it attached again!
Make another leg!

- Re-initializing the emitter and using the CZ gate, we can get it attached again!
- Isn’t this fun? Do another bunch of pumps and Hadamards
Make another leg!

- Re-initializing the emitter and using the CZ gate, we can get it attached again!
- Isn’t this fun? Do another bunch of pumps and Hadamards
- I’ll skip the gore. Do this a few more times and then disconnect the emitter.
This “star” sub-graph looks like

\[ |0\rangle|+\rangle|+\rangle|+\rangle|+\rangle|+\rangle|+\rangle + |1\rangle|\rangle|\rangle|\rangle|\rangle|\rangle|\rangle \]
“Local Complementation”

- This “star” sub-graph looks like

\[ |0\rangle|+\rangle|+\rangle|+\rangle|+\rangle|+\rangle|+\rangle + |1\rangle|\rangle|\rangle|\rangle|\rangle|\rangle|\rangle \]

- Rotate into $y$ basis ($\pi/2$ around $X$ axis for ancilla, $Z$ axis for others)

\[ |+y; +y; +y; +y; +y; +y; +y\rangle + |−y; −y; −y; −y; −y; −y; −y\rangle \]

- Left as exercise to see this is the complete graph
• Measure the ancilla

• This particular state is the 6-legged Azuma repeater graph state (RGS)

• We can “route” through this state
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• Supposed we wanted to connect two legs, even if the other legs had something happen to them.
Repeater Graph State

- Measure the ancilla
- This particular state is the 6-legged Azuma repeater graph state (RGS)
- We can “route” through this state
- Supposed we wanted to connect two legs, even if the other legs had something happen to them.
- Measure the red circles in $z$ basis.
Repeater Graph State

- Measure the ancilla
- This particular state is the 6-legged Azuma repeater graph state (RGS)
- We can “route” through this state
- Supposed we wanted to connect two legs, even if the other legs had something happen to them.
- Measure the red circles in $z$ basis.
- Measure the remaining (uncircled) qubits in $x$ basis.

\[
|0\rangle|+\rangle|+\rangle + |1\rangle|\rangle|\rangle = |+\rangle \left[ |+\rangle|+\rangle + |\rangle|\rangle \right] + |+\rangle \left[ |+\rangle|+\rangle - |\rangle|\rangle \right]
\]
Entanglement with Quantum Emitters: Physical Systems
Nitrogen Vacancies in Diamond

- Diamond lattice, knock out two adjacent carbons (valence 4)
- Put back a nitrogen “N” (valence 5)
- That’s an extra electron! Goes into the other spot “V”
- This impurity electron has properties distinct from the bulk electrons; it’s optically active! “color center.”
- Actually, want an extra electron: “NV−” (e.g. gate the site). 6 $e^-$ coordinate to create $|S| = 1$ object.
The Energy Levels of the NV$^-$

- Two states with $|S_z| = 1$
- Extraordinarily stable: spin-coherence time $\sim 1$ ms at room temperature!
- Ground-state $|S_z| = 0$ admits microwave manipulations to $|S_z| = 1$ states
- Excited states (higher orbital states) are separated by optical transitions
Entangled Photons

- Optical transitions are polarization selected
- $x$-polarized light drives excitation

\[
[L + R] [|1\rangle + |\bar{1}\rangle] \\
\rightarrow |L\rangle |1\rangle + |R\rangle |\bar{1}\rangle + |E_{+}, 1\rangle + |E_{-}, \bar{1}\rangle
\]

- Contingent on light being emitted:

\[
|E_{+}, 1\rangle + |E_{-}, \bar{1}\rangle \\
\rightarrow |1\rangle |R\rangle + |\bar{1}\rangle |L\rangle
\]

$T_1 \sim 10\mu s$

- Photon-NV entanglement!
Entangled Nuclear “ancilla”

- Can we get entanglement with a nuclear ancilla? Let's try to make a CZ gate

- Drive $|0 \downarrow\rangle \leftrightarrow |1 \downarrow\rangle$ with $\pi$ phase.

- Driving this transition can also drive other, nearby harmful transitions. I.e., $|0 \uparrow\rangle \leftrightarrow |1 \uparrow\rangle$

- Hyperfine splitting gives $A \sim 50$ kHz shift. Energy-time resolution requires $\sim 660 \mu s$ to resolve this.

- Entanglement. But can we do this faster?

\[
|E_+\rangle |\uparrow\rangle \quad |E_-\rangle |\downarrow\rangle \\
|E_+\rangle |\downarrow\rangle \quad |E_-\rangle |\uparrow\rangle \\
|1 \uparrow\rangle \quad |1 \downarrow\rangle \\
|\bar{1} \uparrow\rangle \quad |\bar{1} \downarrow\rangle \\
|E_+\rangle |\uparrow\rangle \quad |E_-\rangle |\downarrow\rangle \\
|E_+\rangle |\downarrow\rangle \quad |E_-\rangle |\uparrow\rangle \quad E_{ZFS} \quad A \quad |0 \uparrow\rangle \quad |0 \downarrow\rangle
Can we entangle with a nuclear ancilla? (make a CZ)

Apply $4\pi$ pulse to $|0 \uparrow\rangle \leftrightarrow |1 \uparrow\rangle$

The harmful transition, therefore is \textit{not} driven (only relative phase matters)

Carefully shaping the pulse $\Omega(t) = \Omega_0 \text{sech}(\sigma t)$, we get the second graph: about tenfold improvement in speed.

Fast entanglement with nuclear spin!\textsuperscript{1}

Self assembled Quantum Dots

- “Particle in a box” electron with $J_z = +1/2, +3/2$ at left (opposite angular momentum at right).
- Selection rules prohibit cross-excitation or decay via light.
- Optical pumping (orange line) excites electron, with spontaneous decay in $\sim 1\text{ns}$.
- Polarization depends on angular momentum of electron!
• Gershoni 2016: 1D cluster state, demonstrated 3 to 5 photon cluster state \(^1\)
• Long-lived dark exciton state are excited into optically active biexciton
• Polarization of spontaneous emitted photon depends on state of quantum dot

\(^1\)I. Schwartz et al., Science 354, 434–437 (2016)
Quantum Dots “Molecules”

- Grow stacked quantum dots, each is optically active
- Different energy levels allows individual pumping of each dot
- Exchange interaction creates “always-on” entanglement source
- Produce $cz$ gate between dots\(^1\)

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\(^1\)M. Gimeno-Segovia et al., 2018
• Pairs of quantum dots can form a 4-legged RGS
• Only one entangling operation required
• Coherence time $T_2 \sim 1\mu s$
• Entangling gate $\sim 20\text{ns}$ ($J \sim 5\text{GHz}$)

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• Trapped $^{40}\text{Ca}^+$ ion

• In a cavity, with Zeeman splitting the $S$ and $S'$ (and $D$ and $D'$ states).

• Driving (blue lines) excite the system out of the $S$ manifold.

• Spontaneous emission would, in free space, be circularly or $\pi$-polarized, but in the cavity are projected into horizontal and vertical polarizations.

• Coherent manipulations (green lines) are used to return the state from the metastable $D$ manifold to the stable $S$ manifold.
Long term: 2d cluster

- Use only nearest-neighbor connectivity
- Parallelizable
- Consume the graph state (measure) before the whole sheet is produced
- Generalizes to higher dimensions (provides fault-tolerance)

\[^{1}\text{A. Russo et al., arXiv:1811.06305v1 (2018)}\]
Thank you for your attention!

- Quantum entangled graph states are very useful
- Are tractable to work with
- Quantum emitters can produce them
  (NV centers, quantum dots, trapped ions)