Hagedorn transition in a string bit model[1,2]

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Miami 2017 Conference

[1] S. Raha, Phys. Rev. D 96, 086006 (2017)

[2] T.L. Curtright, S. Raha and C.B. Thorn, Phys. Rev. D 96, 086021 (2017)

Outline

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- (Canonical) Partition function diverges if degeneracy is exponential in energy.

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- Color deconfinement transition.
- Emergence of space.

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- Quartic term $\propto tr\left[\bar{a}\bar{a}aa\right]$, $tr\left[\bar{a}\bar{b}ba\right]$, etc.

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- How to achieve this?⟶Use group characters.

SU(N) characters

- How many singlets from a tensor product of (any number of) adjoints?[3]

[3] T.L. Curtright and C.B. Thorn, Nuclear Physics B 274 (1986).

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- Assign a color character to each creation operator. eg.

$$\bar{a}_l^k |0\rangle \quad \xrightarrow{\chi} \quad \exp\{i\,(\theta_k - \theta_l)\}$$

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- Characters of irreps satisfy orthonormal relations:

$$\frac{\int_G dg\, \bar{\chi}_R(g)\, \chi_{R'}(g)}{\int_G dg} = \begin{cases} \delta_{R,R'} & R' \text{is an irrep} \\ g_{R,R'} & R' \text{is reducible} \end{cases}$$

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- Multiplicity of any irrep in any reducible representation.

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Generating functions

- Generating function for characters to generating functions for multiplicities.

$$\sum_{M=0}^{\infty} \chi(M)\, x^M \quad \xrightarrow{\bar{\chi}_R} \quad \sum_{M=0}^{\infty} g_R(M)\, x^M \xrightarrow[x=e^{-\beta\omega}]{} Z(\beta)$$

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- Once you do everything:

$$Z_N = \frac{\int_0^{2\pi} \prod_k d\theta_k \prod_{i<j} 4\sin^2\left(\frac{\theta_i-\theta_j}{2}\right) \left\{\frac{1-x}{1+x} \prod_{k,l} \frac{1+xe^{i(\theta_k-\theta_l)}}{1-xe^{i(\theta_k-\theta_l)}}\right\}}{\int_0^{2\pi} \prod_k d\theta_k \prod_{i<j} 4\sin^2\left(\frac{\theta_i-\theta_j}{2}\right)}$$

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- The denominator is $N!(2\pi)^N$.

For SU(2)

- Character generating function:

$$\left(\frac{1+x}{1-x}\right)\frac{1+x^2+2x\cos(\theta_1-\theta_2)}{1+x^2-2x\cos(\theta_1-\theta_2)}$$
$$=1+(4\cos(\theta_1-\theta_2)+2)\,x+2\,(2\cos(\theta_1-\theta_2)+1)^2\,x^2+\cdots$$

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- Basis states:

$$\left\{\begin{array}{l} tr\left[\bar{a}^2\right]|0\rangle \\ tr\left[\bar{a}\bar{b}\right]|0\rangle \end{array}\right., \left\{\begin{array}{l} tr\left[\bar{a}^3\right]|0\rangle \\ tr\left[\bar{a}^2\bar{b}\right]|0\rangle \end{array}\right., \text{ etc.}$$

Analytic results

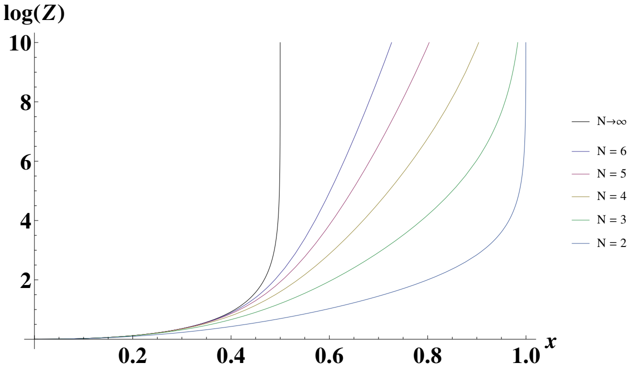

Figure 1: Currently known partition functions

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[4] S. Raha, Phys. Rev. D 96, 086006 (2017)

Outline

High temperature limit

$$\lim_{x \to 1} Z = \left(\frac{2}{1-x} \right)^{N-1} \frac{1}{N!} \frac{\int \cdots \int_0^{2\pi} \left(\prod_k d\theta_k \right) \left\{ \prod_{i<j} 4 \cos^2 \left(\frac{\theta_i - \theta_j}{2} \right) \right\}}{(2\pi)^N}$$

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- E.g. the coefficients of:

$$z_1 z_2 \text{in} \, (z_1+z_2)^2$$
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- Can you find R_3?

Estimates of R_N

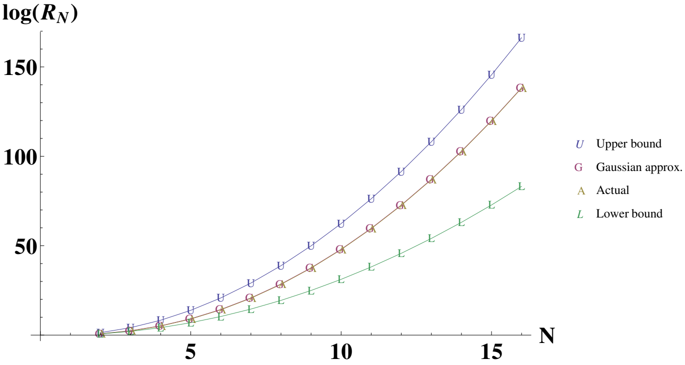

Figure 2: Different estimates of R_N

[5] B.D. McKay, Congressus Numerantium 40, (1983)

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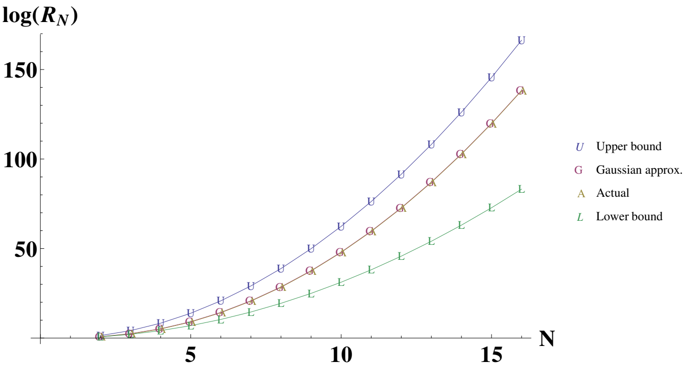

Figure 2: Different estimates of R_N

R_N enumerates Eulerian digraphs with N nodes![5]

[5] B.D. McKay, Congressus Numerantium 40, (1983)

Eulerian digraphs

- Edges have directions.

[6] B.D. McKay, Combinatorica 10, (1990)
[7] T.L. Curtright, S. Raha and C.B. Thorn, Phys. Rev. D 96, 086021 (2017)

Eulerian digraphs

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- Traverse every edge exactly once.

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- McKay[6] estimated $R_N = \left(\frac{2^N}{\sqrt{\pi N}}\right)^{N-1} e^{-1/4}\sqrt{N}\left\{1+\mathscr{O}\left(\frac{1}{\sqrt{N}}\right)\right\}$

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- Including only quartic terms $\sum_{j<k}\left(\theta_j - \theta_k\right)^4$ in the exponent.

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- Curtright[7] incorporated terms till $\sum_{j<k}(\theta_j - \theta_k)^6$
  $R_N = \left(\frac{2^N}{\sqrt{\pi N}}\right)^{N-1} e^{-1/4}\sqrt{N}\left\{1 + \frac{3}{16N} + \mathscr{O}\left(\frac{1}{N^2}\right)\right\}$

[6] B.D. McKay, Combinatorica 10, (1990)

[7] T.L. Curtright, S. Raha and C.B. Thorn, Phys. Rev. D 96, 086021 (2017)

Outline

Steepest descent

$$\log(Z) = \log\left[\int\cdots\int_0^{2\pi}\left(\prod_k d\theta_k\right)e^{L(\{\theta\})}\right] + (N-1)\log\left(\frac{1+x}{1-x}\right) - \log\left(N!\,(2\pi)^N\right)$$

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where $L(\{\theta\}) = \log\left\{\prod_{i<j} 4\sin^2\left(\frac{\theta_i-\theta_j}{2}\right)\frac{1+x^2+2x\cos(\theta_i-\theta_j)}{1+x^2-2x\cos(\theta_i-\theta_j)}\right\}$

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$$\log(Z) = \overbrace{\log(L(\{\theta_{\max}\})) + (N-1)\log\left(\frac{1+x}{1-x}\right)}^{f^{[\max]}} + \log\left[\int \cdots \int_0^{2\pi} \left(\prod_k d\theta_k\right) e^{\frac{\theta^2}{2}\frac{d^2L}{d\theta^2}|_{\theta_{\max}} + \cdots}\right] - \log\left(N!\,(2\pi)^N\right)$$

Leading behaviour

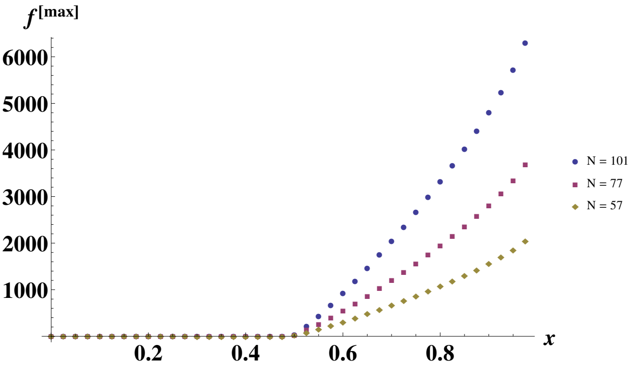

Figure 3: Leading approximation for the partition function

Leading behaviour

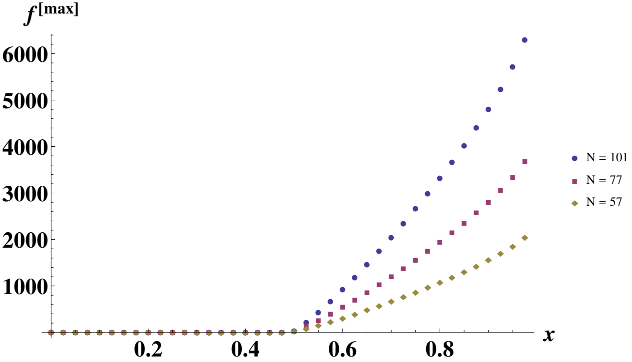

Figure 3: Leading approximation for the partition function

$$f^{[max]} = c_1 N^2 + c_2 N \log(N) + c_3 N + c_4 \log(N) + c_5 + \mathcal{O}\left(\frac{1}{N}\right)$$

Order parameter

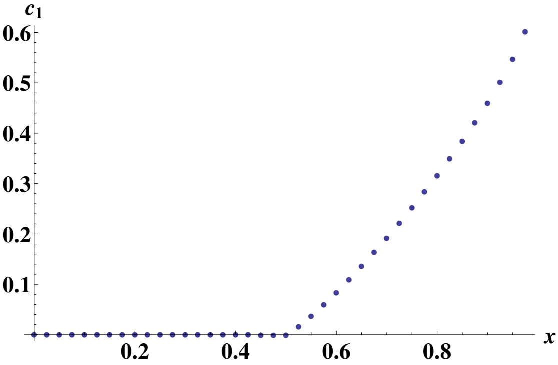

Figure 4: The coefficient of N^2 switches on at $x = 1/2$

Distribution of θ_{max}'s

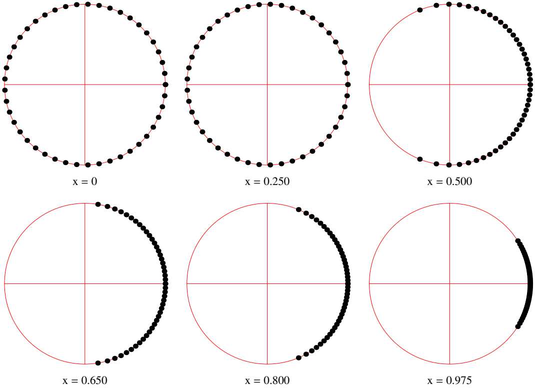

Figure 5: Relative positions of θ_{max}'s as a function of temperature for $N = 45$

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[8] E. Brézin, C. Itzykson, G. Parisi, and J. B. Zuber, Communications in Mathematical Physics 59, (1978)

[9] David J. Gross and Edward Witten, Physical Review D 21, (1980)

Density function

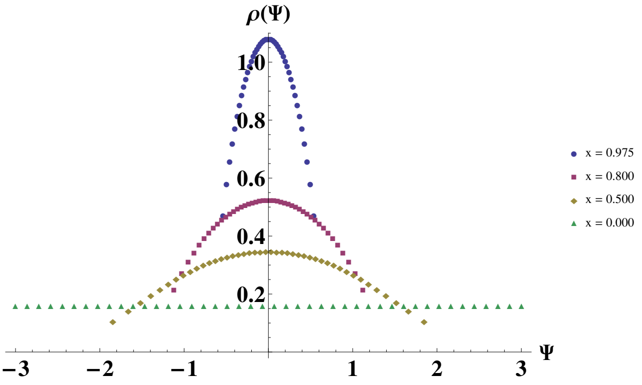

Figure 6: The normalized probability density function of θ_{max} for $N = 45$

1011

[10] B. Sundborg, Nuclear Physics B 573, (2000)

[11] O. Aharony, J. Marsano, S. Minwalla, K. Papadodimas, and M.V. Raamsdonk, Advances in Theoretical and Mathematical Physics 8, (2004).

Effective Field Theory

- From an earlier slide:

$$\log(Z) = \log\left[\int \cdots \int_0^{2\pi} \left(\prod_k d\theta_k\right) e^{L(\{\theta\})}\right] + (N-1)\log\left(\frac{1+x}{1-x}\right) - \log\left(N!\,(2\pi)^N\right)$$

[12] T.L. Curtright, S. Raha and C.B. Thorn, Phys. Rev. D 96, 086021 (2017)

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- For finite N one can shift the singularity off the real axis by doing a partial sum to all orders.

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Thank you!

(Did you figure out the value of R_3?)