The Viability of Phantom Dark Energy as a Quantum Field in 1st-Order FLRW Space

Kevin J. Ludwick

LaGrange College

December 15, 2017

Discovery of Dark Energy

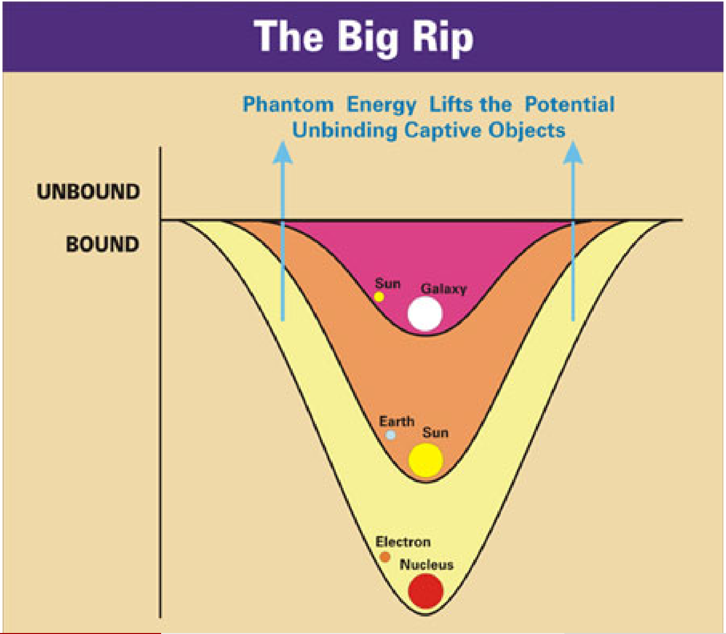
- High-z Supernova Search Team in 1998, Supernova Cosmology Project in 1999: SNIa spectra
- Conclusion: dark energy, responsible for cosmic acceleration
- Other evidence: galaxy surveys, late-time integrated Sachs-Wolfe effect (evidence for the effect of dark energy on superclusters and supervoids in the CMB)
- 2011 Nobel Prize: Schmidt, Riess, Perlmutter

Characteristics of Dark Energy

- About 68% of our universe is dark energy
- Physical intuition of the nature and dynamics of DE lacking
- Strange feature: as volume increases (i.e., universe expands), DE density decreases at lower rate compared to that of normal matter
- DE density can even stay constant or increase as volume increases

Modeling Dark Energy

- Relationship between pressure and density usually assumed to be $p_i = w_i \rho_i$
- For the cosmological constant (CC) model, $w_\Lambda = -1$, and this gives constant DE density
- $-1 \leq w_{DE} < -1/3$: quintessence dark energy (density decreases or stays constant as universe expands)
- $w_{DE} < -1$: phantom dark energy (density increases as universe expands; rate of acceleration increases; leads to a rip)


The Big Rip
Phantom Energy Lifts the Potential
Unbinding Captive Objects
UNBOUND
BOUND
Sun
Galaxy
Earth
Sun
Electron
Nucleus

Observational Constraints

- Planck 2015: $w = -1.006 \pm 0.045$
- Planck 2013: $w = -1.13^{+0.13}_{-0.10}$
- WMAP9 (CMB+BAO+H_0+SNIa): $w = -1.084 \pm 0.063$
- Suggestive that dark energy really could be phantom

Scalar Field Dark Energy

- Using the flat FLRW metric: $ds^2 = a^2(\tau)[-d\tau^2 + dx^i dx_i]$
- $S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right] + S_m$
- homogeneous: $\rho_\phi = \frac{\dot{\phi}^2}{2a^2} + V(\phi), \quad P_\phi = \frac{\dot{\phi}^2}{2a^2} - V(\phi)$
- $w_\phi \geq -1 \iff \rho_\phi + P_\phi \geq 0 \iff KE_\phi \text{ term} = \frac{\dot{\phi}^2}{2a^2} \geq 0$

Phantom DE: $w < -1$

Phantom DE: $w < -1$

$w < -1$ implies negative KE term:

$$\rho_\phi = \frac{\dot{\phi}^2}{2a^2} + V(\phi), \quad P_\phi = \frac{\dot{\phi}^2}{2a^2} - V(\phi)$$

$$w_\phi < -1 \iff \rho_\phi + P_\phi < 0 \iff KE_\phi \text{ term} = \frac{\dot{\phi}^2}{2a^2} < 0$$

Phantom DE: $w < -1$

$w < -1$ implies negative KE term:

$$\rho_\phi = \frac{\dot{\phi}^2}{2a^2} + V(\phi), \quad P_\phi = \frac{\dot{\phi}^2}{2a^2} - V(\phi)$$

$$w_\phi < -1 \iff \rho_\phi + P_\phi < 0 \iff KE_\phi \text{ term} = \frac{\dot{\phi}^2}{2a^2} < 0$$

NOT GOOD

- As a result, what is usually done for phantom field ($w_\phi < -1$): sign flip in front of the kinetic term in the action so KE_ϕ term is positive
- When this is done, the phantom field is ghostlike: phantom DE can decay to a potentially unlimited number of heavier, more energetic particles (i.e., gravitons) along with DE particles of negative energy!

- As a result, what is usually done for phantom field ($w_\phi < -1$): sign flip in front of the kinetic term in the action so KE_ϕ term is positive
- When this is done, the phantom field is ghostlike: phantom DE can decay to a potentially unlimited number of heavier, more energetic particles (i.e., gravitons) along with DE particles of negative energy!
- Effective field theory *may* be able to render this instability unobservable, but not without great difficulty. But perhaps there's a simpler way....

Alternative, Accurate Framework

$$\phi(\tau) \to \phi(\tau) + \delta\phi(\vec{x}, \tau)$$

$$w_{eff} \equiv \frac{P_\phi + \delta P_\phi}{\rho_\phi + \delta\rho_\phi} = \frac{\frac{1}{2a^2}(\dot{\phi}^2 + 2\dot{\phi}\dot{\delta\phi}) - (V(\phi) + V'(\phi)\delta\phi)}{\frac{1}{2a^2}(\dot{\phi}^2 + 2\dot{\phi}\dot{\delta\phi}) + (V(\phi) + V'(\phi)\delta\phi)}$$

$$\mathrm{KE}_{eff} = \frac{1}{2a^2}(\dot{\phi}^2 + 2\dot{\phi}\dot{\delta\phi})$$

For $w_\phi < -1$, $\rho_\phi + P_\phi < 0$, but still possible for $w_{eff} \geq -1$:

$$\rho_\phi + \delta\rho_\phi + P_\phi + \delta P_\phi \geq 0 \iff \mathrm{KE}_{eff} \geq 0 \iff w_{eff} \geq -1$$

Perhaps only the *full* perturbed phantom fluid is the true phantom DE field, $\Phi(\vec{x},\tau)$:

$$\rho_\Phi(\vec{x},\tau) \equiv \rho_\phi(\tau) + \delta\rho_\phi(\vec{x},\tau) = -\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + V(\Phi), \quad (1)$$

$$P_\Phi(\vec{x},\tau) \equiv P_\phi(\tau) + \delta P_\phi(\vec{x},\tau) = -\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - V(\Phi), \quad (2)$$

$$2\mathrm{KE}_\Phi = \rho_\Phi + P_\Phi = -g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi, \quad (3)$$

So for an apparent value of $w_{DE} < -1$ as measured by observational probes, it may be the case that $w_\Phi \equiv P_\Phi/\rho_\Phi \geq -1$ and $\mathrm{KE}_\Phi \geq 0$, indicative of a viable scalar field theory for phantom dark energy.

Cosmological Perturbation Theory

synchronous gauge: $ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$

$$h_{ij}(\vec{x},\tau) = \int d^3k e^{i\vec{k}\cdot\vec{x}}\left\{\hat{k}_i\hat{k}_j h(\vec{k},\tau) + (\hat{k}_i\hat{k}_j - \frac{\delta_{ij}}{3})6\eta(\vec{k},\tau)\right\}$$

Perturbed stress-energy tensor:

$$\begin{aligned} T^0{}_0 &= -(\rho + \delta\rho), \\ T^0{}_i &= (\rho + P)v_i, \\ T^i{}_j &= (P + \delta P)\delta^i{}_j + \Sigma^i{}_j, \quad \Sigma^i{}_i = 0. \end{aligned}$$

Solve perturbed Einstein's equation (1st-order part): $\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$ Solve conservation of energy and momentum (1st-order part): $\delta T^{\mu\nu}{}_{;\mu} = 0$

Relevant Length Scales

- Type Ia supernovae for DE detection:
  $z \approx 0.3$ to $z \approx 2$
- late-time Sachs-Wolfe effect: similarly large scales
- An acceptable theory of phantom DE must be valid for at least this range of large scales.
- We use this range in our analysis, and we use Planck data to constrain our constant of integration in our radiation-era perturbations, then specify the constant of integration in the following eras by continuity.

Constant $w < -1$, DE Domination

$$\rho_\phi + \delta\rho_\phi + P_\phi + \delta P_\phi \geq 0 \text{ for positive KE implies}$$

$$-2^{\frac{7}{2}+\frac{1}{1+3w}} 3^{-\frac{1}{2}-\frac{1}{1+3w}} a^{\frac{1}{2}(-3-3w)} \pi^{\frac{3}{2}+\frac{1}{1+3w}} \left(\frac{1}{r^2}\right)^{1-\frac{1}{1+3w}} S(3+3w) \rho_{DE0}^{\frac{1}{2}+\frac{1}{1+3w}} \cos\left[\frac{\pi}{1+3w}\right] \Gamma\left[1-\frac{2}{1+3w}\right]$$

is ≤ -1. It turns out that the above expression is the same thing as $\delta_\phi \equiv \delta\rho_\phi/\rho_\phi$, and this perturbation, by definition, cannot be ≤ -1.

So KE is negative for constant $w < -1$ during DE domination, regardless of value of constant of integration S.

Conclusion

Phantom DE: **Possible to have positive KE for non-constant w for some relevant ranges of length and time in 1st-order perturbation theory, but not for all.**

Quintessence DE: **Possible to have negative KE for some ranges of length and time, specifically for (constant w) DM-DE era in 1st-order perturbation theory for constant $w \geq -1$. We suspect the same for non-constant parametrizations.**
For more details: K.L., Phys. Rev. D 92, 063019 (2015) (arXiv:1507.06492)

Side note:

Constant $w_\phi = -1$: It turns out that the relevant perturbations for determining the kinetic energy are 0, and it always has positive KE in 1st-order perturbation theory.

Conclusion

So we see that phantom and quintessence DE as a basic single-field theory may not categorically have positive and negative KE term, respectively.

Quantum Treatment

- We want to check the kinetic term still: $-\frac{1}{2}g^{\sigma\rho} < \partial_\rho\phi\partial_\sigma\phi >= -\frac{1}{2}g^{\sigma\rho}\partial_\rho\partial'_\sigma iG(x,x')|_{x'\to x}$
- We solve $(-\Box_x + m^2 + \xi R^2)G(x,x') = g^{-1/4}(x)\delta(x-x')g^{-1/4}(x')$ by using Riemann normal coordinates:

Quantum Treatment

- We want to check the kinetic term still: $-\frac{1}{2}g^{\sigma\rho} < \partial_\rho\phi\partial_\sigma\phi >= -\frac{1}{2}g^{\sigma\rho}\partial_\rho\partial'_\sigma iG(x,x')|_{x'\to x}$
- We solve $(-\Box_x + m^2 + \xi R^2)G(x,x') = g^{-1/4}(x)\delta(x-x')g^{-1/4}(x')$ by using Riemann normal coordinates:
- Assuming for any point P in the neighborhood of Q that there is a unique geodesic joining these points, we can use Riemann normal coordinates of that point P: $y^\mu = \lambda\xi^\mu$, where ξ^μ is the tangent to the geodesic at the point Q, and λ is a parameter representing how far off along the geodesic we are from Q. We take the origin Q at the spacetime point x', where $y^\mu = 0$, and we denote x^μ to be its normal coordinate y^μ.

Quantum Treatment

- We want to check the kinetic term still: $-\frac{1}{2}g^{\sigma\rho} < \partial_\rho\phi\partial_\sigma\phi >= -\frac{1}{2}g^{\sigma\rho}\partial_\rho\partial'_\sigma iG(x,x')|_{x'\to x}$
- We solve $(-\Box_x + m^2 + \xi R^2)G(x,x') = g^{-1/4}(x)\delta(x-x')g^{-1/4}(x')$ by using Riemann normal coordinates:
- Assuming for any point P in the neighborhood of Q that there is a unique geodesic joining these points, we can use Riemann normal coordinates of that point P: $y^\mu = \lambda\xi^\mu$, where ξ^μ is the tangent to the geodesic at the point Q, and λ is a parameter representing how far off along the geodesic we are from Q. We take the origin Q at the spacetime point x', where $y^\mu = 0$, and we denote x^μ to be its normal coordinate y^μ.
- We can now write $g_{\mu\nu}(y=0) = \eta_{\mu\nu}$ and $|g(y=0)| = 1$ and expand $g_{\mu\nu}(y)$, $R(y)$, and $g(y)$ about $y^\mu = 0$. For example, $g_{\mu\nu}(y) = \eta_{\mu\nu} - \frac{1}{3}R_{\mu\alpha\nu\beta}(0)y^\alpha y^\beta - \frac{1}{6}R_{\mu\alpha\nu\beta;\gamma}(0)y^\alpha y^\beta y^\gamma + \ldots$
- So now, $x' \to x$ is equivalent to $y \to 0$

Adiabatic Subtraction

- Defining $G(x,x') = g(x)^{-1/4}\bar{G}(x,x')g(x')^{-1/4}$ and making the Fourier transform $\bar{G}(x,x') = \int \frac{d^n k}{(2\pi)^n} e^{iky}\bar{G}(k)$, we express $\bar{G}(k) = \bar{G}_0(k) + \bar{G}_1(k) + \bar{G}_2(k) + ...$, where $\bar{G}_i(k)$ involves i derivatives of the metric and is of order $k^{-(2+i)}$
- For a given interval (say from x to x'), our adiabatic assumption is that the rate of change of $a(t)$ is sufficiently slow, or adiabatic. So each higher-order term in metric derivative should be smaller than the previous.

Adiabatic Subtraction

- Defining $G(x,x') = g(x)^{-1/4}\bar{G}(x,x')g(x')^{-1/4}$ and making the Fourier transform $\bar{G}(x,x') = \int \frac{d^n k}{(2\pi)^n} e^{iky}\bar{G}(k)$, we express $\bar{G}(k) = \bar{G}_0(k) + \bar{G}_1(k) + \bar{G}_2(k) + \ldots$, where $\bar{G}_i(k)$ involves i derivatives of the metric and is of order $k^{-(2+i)}$
- For a given interval (say from x to x'), our adiabatic assumption is that the rate of change of $a(t)$ is sufficiently slow, or adiabatic. So each higher-order term in metric derivative should be smaller than the previous.
- It turns out we need to solve iteratively at least up to $i = 5$ in 4 dimensions in order to subtract out all ultraviolet divergences and keep some finite part from $KE = -\frac{1}{2}g^{\sigma\rho} < \partial_\rho\phi\partial_\sigma\phi >= -\frac{1}{2}g^{\sigma\rho}\partial_\rho\partial'_\sigma iG(x,x')|_{x'\to x}$.

Adiabatic Subtraction

- Defining $G(x,x') = g(x)^{-1/4}\bar{G}(x,x')g(x')^{-1/4}$ and making the Fourier transform $\bar{G}(x,x') = \int \frac{d^n k}{(2\pi)^n} e^{iky}\bar{G}(k)$, we express $\bar{G}(k) = \bar{G}_0(k) + \bar{G}_1(k) + \bar{G}_2(k) + \ldots$, where $\bar{G}_i(k)$ involves i derivatives of the metric and is of order $k^{-(2+i)}$
- For a given interval (say from x to x'), our adiabatic assumption is that the rate of change of $a(t)$ is sufficiently slow, or adiabatic. So each higher-order term in metric derivative should be smaller than the previous.
- It turns out we need to solve iteratively at least up to $i = 5$ in 4 dimensions in order to subtract out all ultraviolet divergences and keep some finite part from $KE = -\frac{1}{2}g^{\sigma\rho} < \partial_\rho\phi\partial_\sigma\phi >= -\frac{1}{2}g^{\sigma\rho}\partial_\rho\partial'_\sigma iG(x,x')|_{x'\to x}$.
- Using adiabatic subtraction, we can treat $KE_{phys} = \sum KE_{all\ \ orders} - \sum KE_{divergent} \approx KE_{i=4\ \ and\ \ i=5}$

Kinetic Energy via Adiabatic Subtraction

- With this Fourier expression for $G(x,x')$, $\partial/\partial y^\alpha \to ik_\alpha$ and $y^\alpha \to i\partial/\partial k_\alpha$ acting on $\bar{G}_i(k)$
- So using all this in $(-\Box_x + m^2 + \xi R(x)^2)G(x,x') = g^{-1/4}(x)\delta(x-x')g^{-1/4}(x')$, expanding about $y^\mu = 0$, we can solve iteratively for each $\bar{G}_i(k)$.
- $\bar{G}_0(k) = (k^2+m^2)^{-1}$
- $\bar{G}_1(k) = (\frac{1}{6} - \xi)R(0)(k^2+m^2)^{-2}$
- ... and so on to 5th order, with coefficients evaluated at $y = 0$

Kinetic Energy via Adiabatic Subtraction

- With this Fourier expression for $G(x,x')$, $\partial/\partial y^\alpha \to ik_\alpha$ and $y^\alpha \to i\partial/\partial k_\alpha$ acting on $\bar{G}_i(k)$
- So using all this in $(-\Box_x + m^2 + \xi R(x)^2)G(x,x') = g^{-1/4}(x)\delta(x-x')g^{-1/4}(x')$, expanding about $y^\mu = 0$, we can solve iteratively for each $\bar{G}_i(k)$.
- $\bar{G}_0(k) = (k^2+m^2)^{-1}$
- $\bar{G}_1(k) = (\frac{1}{6}-\xi)R(0)(k^2+m^2)^{-2}$
- ... and so on to 5th order, with coefficients evaluated at $y = 0$
- $KE = -\frac{1}{2}g^{\sigma\rho}\partial_\rho\partial'_\sigma iG(x,x')|_{x'\to x} = -\frac{i}{2}g^{\sigma\rho}(y)\frac{\partial}{\partial x^\rho}\frac{\partial}{\partial x^{\sigma\prime}}\int\frac{d^4k}{(2\pi)^4}e^{iky}\bar{G}(k,x')|_{y\to 0}$
- $\partial/\partial x^\rho \to ik_\rho$ inside the integral, and $\partial/\partial x^{\sigma\prime}$ acts on the coefficients evaluated at $y=0$ in $\bar{G}(k,x')$, and we just have $i=4$ and $i=5$. (Turns out the contribution from $i=4$ integrates to 0.)

Kinetic Energy via Adiabatic Subtraction

- With this Fourier expression for $G(x,x')$, $\partial/\partial y^\alpha \to ik_\alpha$ and $y^\alpha \to i\partial/\partial k_\alpha$ acting on $\bar{G}_i(k)$
- So using all this in $(-\Box_x + m^2 + \xi R(x)^2)G(x,x') = g^{-1/4}(x)\delta(x-x')g^{-1/4}(x')$, expanding about $y^\mu = 0$, we can solve iteratively for each $\bar{G}_i(k)$.
- $\bar{G}_0(k) = (k^2+m^2)^{-1}$
- $\bar{G}_1(k) = (\frac{1}{6} - \xi)R(0)(k^2+m^2)^{-2}$
- ... and so on to 5th order, with coefficients evaluated at $y = 0$
- $KE = -\frac{1}{2}g^{\sigma\rho}\partial_\rho\partial'_\sigma iG(x,x')|_{x'\to x} = -\frac{i}{2}g^{\sigma\rho}(y)\frac{\partial}{\partial x^\rho}\frac{\partial}{\partial x^{\sigma\prime}}\int \frac{d^4k}{(2\pi)^4}e^{iky}\bar{G}(k,x')|_{y\to 0}$
- $\partial/\partial x^\rho \to ik_\rho$ inside the integral, and $\partial/\partial x^{\sigma\prime}$ acts on the coefficients evaluated at $y = 0$ in $\bar{G}(k,x')$, and we just have $i = 4$ and $i = 5$. (Turns out the contribution from $i = 4$ integrates to 0.)
- Then applying limit $y \to 0$ gives $e^{iky} \to 1$, and all the remaining k-dependence is $\int \frac{d^4k}{(2\pi)^4}(k^2+m^2)^{-3} = \frac{i}{32\pi^2 m^2}$

Plug in 1st-Order FLRW Metric

- Expression for KE: too long to show
- We use the 1st-order FLRW metric, $ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$, assuming DE domination and using those solutions for the perturbations for constant w. After a LENGTHY calculation (with the help of the xTensor package for Mathematica), we arrive at an analytic expression for KE dependent on scale factor a, radial distance r, w, mass m, non-minimal coupling ξ, and the constant of integration S mentioned earlier.

KE Plot (from start of DE domination to $a = 10$)

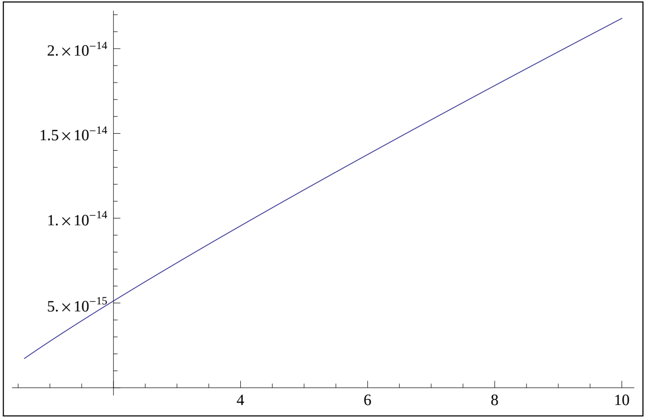


Figure : KE vs. a for $r = 5412$ Mpc (obtained from distance along null geodesic from $z = 2$), $m = 3.2 \times 10^{-34}$ eV, $\xi = 0$, $w = -1.1$, $S = 10^{-5}$. Looks virtually the same when evaluated at $r = 1252$ Mpc (obtained from distance along null geodesic from $z = 0.3$). KE scales up as mass lowered.

KE vs S

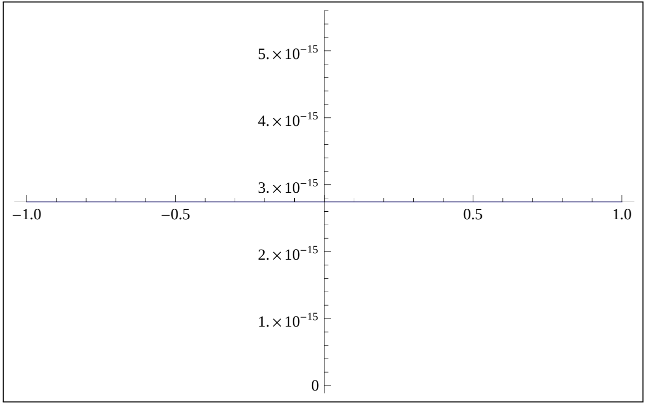


Figure : KE vs. S for $a = 1$ (present day), $r = 5412$ Mpc (obtained from distance along null geodesic from $z = 2$), $m = 3.2 \times 10^{-34}$ eV, $\xi = 0$, $w = -1.1$. From matching with data, S is expected be small, and KE unaffected by its size, for any a value. So mainly due to 0th order.

KE vs w

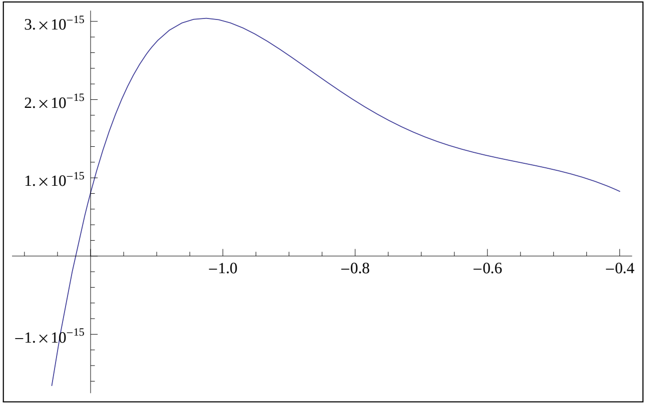


Figure : KE vs. w for $a = 1$ (present day), $r = 5412$ Mpc (obtained from distance along null geodesic from $z = 2$), $m = 3.2 \times 10^{-34}$ eV, $\xi = 0$, $S = 10^{-5}$. KE negative for w a little less than -1.2, and the same holds for a at beginning of DE domination.

KE vs ξ

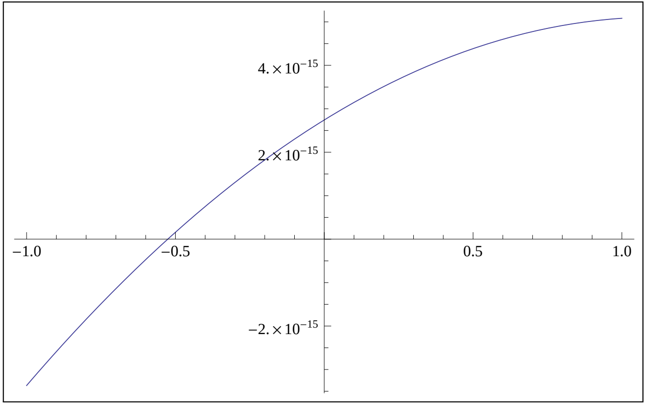


Figure : KE vs. ξ for $a = 1$ (present day), $r = 5412$ Mpc (obtained from distance along null geodesic from $z = 2$), $m = 3.2 \times 10^{-34}$ eV, $w = -1.1$, $S = 10^{-5}$. However, we know that ξ is constrained to be very small.

Summary and Conclusion

- For DE domination, constant $w < -1$, scalar field model ill-defined in FLRW
- Perturbed FLRW (i.e., not perfect homogeneity): condition for positive KE implies the perturbation $|\delta| > 1$
- Finding $<$KE$>$ for DE as a quantum field (constant $w < -1$), approximating the propagator using adiabatic subtraction: positive KE!
- Not shown, but $<$KE$>$ for $w > -1$ also positive
- Can see more details in a paper on the arXiv soon (I hope)