Entanglement in gauge theories and gravity

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1704.07763 + work in progress

December 17, 2017

Plan

The main point of the talk is to suggest an idea for what the RT area might be counting in the bulk and describe the evidence for it.

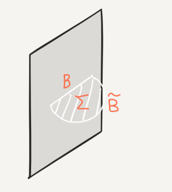
- 10 min Review of RT formula and applications.
- 15 min EE in gauge theories. Conjecture: RT area is the analog of the edge term in the EE of an emergent gauge theory. Qualitative description of ongoing work.

Entanglement and Spacetime

In recent years, people have suggested that "spacetime emerges from quantum entanglement."

All of the mathematically precise work in this direction comes from AdS/CFT and the Ryu-Takayanagi formula,

$$S_{EE}(B) = \frac{A}{4G_N} + S_{EE,bulk}(\Sigma) + \ldots$$



Let me remind you of two applications of it...

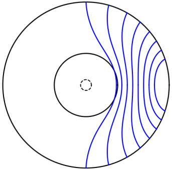
Linearized EFE's from EE 1st law

... around the AdS vacuum. [van Raamsdonk et al.]

$$\begin{array}{ccc} \delta S_{EE} & = & \delta\langle -\log\rho\rangle \\ \updownarrow & & \updownarrow \\ \delta S_{EE} & = & \int_B F(\langle T_{00}(r)\rangle) \\ \updownarrow \text{RT} & & \updownarrow \text{GKPW} \\ \int_{\tilde{B}} F_0(\delta g_{ab}) & = & \int_B F_1(\delta g_{ab}) \end{array}$$

True $\forall\rho$; perturb $\mathrm{Tr}\rho\log\rho$.
For ball regions in vacuum states of CFT's we know the closed form of $\log\rho$.
Use AdS/CFT dictionary
$=$ linearized EFE's.

Can we generalize to getting the linearized Einstein eq.'s around other asymptotically AdS spacetimes?



"Entanglement shadow" in generic horizonless asymptotically-AdS geometries.

A possible resolution: there are more general forms of entanglement. Can these geometrize in the bulk?

Algebraic EE: For $|\psi\rangle \in \mathcal{H}$ and subalgebra $\mathcal{A}_0 \in \mathcal{A}$, $\exists$

$$\rho(= \sum_{\mathcal{O}_i \in \mathcal{A}_0} p_i \mathcal{O}_i) \in \mathcal{A}_0$$

s.t.

$$\mathrm{Tr}_{\mathcal{H}}(\rho\mathcal{O}) = \langle\psi|\mathcal{O}|\psi\rangle \ \forall\ \mathcal{O} \in \mathcal{A}_0\,.$$

Then

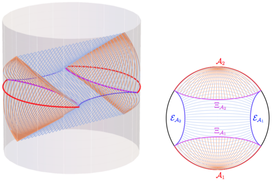
$$S_{EE}(\mathcal{A}_0) = -\mathrm{Tr}\,\rho \log \rho\,.$$

In a previous work, I showed that algebraic EE's can indeed be related to areas of non-minimal surfaces in AdS_3 conical defect backgrounds, but I won't discuss it today.

Entanglement wedge reconstruction

[Dong, Harlow, Wall ...]
In effective field theory on AdS, consider a local bulk operator at a point in the bulk. How much of the boundary CFT do we need to have access to to reconstruct it?

Entanglement wedge:



Any local bulk operator in the entanglement wedge $\mathcal{E}_A$ can be reconstructed as a CFT operator supported on region $\mathcal{A}$!

"RT = entanglement wedge reconstruction".
In fact, Harlow has proved a related theorem for (finite-dimensional) quantum systems...

"RT = entanglement wedge reconstruction" (Harlow)

Harlow's assumptions	AdS/CFT interpretation
$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$	CFT (UV) Hilbert space
Subspace $\mathcal{H}_{IR} \subseteq \mathcal{H}$	"code subspace" of EFT on AdS
Subalgebra $\mathcal{A}_{IR}$ whose action on $\mathcal{H}_{IR}$ keeps us in $\mathcal{H}_{IR}$	gauge-inv. bulk operators.

Then, the following were proved to be mathematically equivalent:

1. $\exists$ subalgebra $\mathcal{A}_{IR,A} \in \mathcal{A}_{IR}$ s.t.
   $\forall\, |\tilde{\psi}\rangle \in \mathcal{H}_{IR}$,
   $\forall\, \tilde{\mathcal{O}} \in \mathcal{A}_{IR,A}$,
   $\exists\, \mathcal{O}_A \in \mathcal{H}_A$ s.t. $\mathcal{O}_A|\tilde{\psi}\rangle = \tilde{\mathcal{O}}|\psi\rangle$.

   Entanglement wedge reconstruction. ($\mathcal{A}_{IR,A}$ = bulk g-inv. operators supported on $\mathcal{E}_A$.)

2. $\exists$ an operator $\mathcal{L}_A$ in $\mathcal{A}_{IR,A} \cap \mathcal{A}_{IR,\bar{A}}$
   s.t. $\forall \rho \in \mathcal{H}_{IR}$,
   $S_{EE}(\rho_A) =$
   $\mathrm{Tr}(\rho \mathcal{L}_A) + S_{alg}(\rho, \mathcal{A}_{IR,A})$.

   RT formula + 1/N correction. ($\mathcal{L}_A$ = RT area.)

End of part 1

To summarize so far,

- The main argument for "entanglement = spacetime" is the Ryu-Takayanagi formula in AdS/CFT.
- Nice consequences include:
  - Steps towards understanding the CFT origin of Einstein's equations.
  - An understanding of subregion duality for bulk operator reconstruction.

I now want to discuss an idea for what the RT area might be counting from the bulk point of view.

Idea

The idea will be to compare EE in an emergent gauge theory to EE in AdS/CFT.

On the left, there is the emergent gauge theory, assuming an explicit (say, a lattice) UV regulator.

	Emergent gauge theory	**AdS/CFT**
UV	Factorizable Hilbert space	CFT
IR	Gauge theory	Effective field theory in AdS
EE	$S_{EE}^{UV}(A) = S_{alg.,ginv}(\mathrm{A}) +$ "$\log d_R$" term	$S_{EE}^{CFT}(A) = \frac{A}{4G_N} + S_{alg.ginv}(\mathcal{E}_A)$

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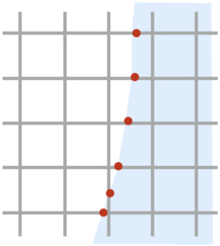
I'll now explain the gauge theory side of the table (before coming back to AdS/CFT).

A proposal for EE in (lattice) gauge theories

I'll first review a completely formal proposal how to define EE in a gauge theory, then argue that it gives the UV answer when the gauge theory is emergent.

In a gauge theory, the Hilbert space doesn't factorize, so we need to get the reduced density matrix in a different way than the usual partial trace.

"Extended Hilbert space" definition: [Buividovich-Polikarpov; Donnelly]

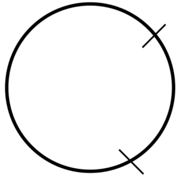


$$\begin{aligned} \mathcal{H} \in \mathcal{H}_{ext.} &= \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}} \\ \rho_A &= \mathrm{Tr}_A \rho \in \mathcal{H}_{ext} \\ S_{EE} &= -\mathrm{Tr} \rho_A \log \rho_A \,. \end{aligned}$$

This definition introduces some "edge terms", including this term that I want to identify with the area. Let me show this through the example of 2d Yang-Mills on a circle.

Ex.: EE in nonabelian gauge theory on $\mathbf{S}^1$

Consider Yang-Mills with gauge group G on the $\mathbf{S}^1$.



- Operator algebra: Wilson loops $\mathrm{Tr}_R \exp(i \oint \mathbf{A})$, Casimirs $E^a E^a, \ldots$
- Hilbert space basis labeled by reps of G: $\{|R\rangle\}$.
- $\mathcal{H}_{ext.} = \oplus\{|R, i, j\rangle \otimes |R, j, i\rangle\}$, $i, j \in 1, \ldots, \dim R$.

For $|\psi\rangle = \sum_R \psi_R |R\rangle \in \mathcal{H}$,

$$\rho_A \in \mathcal{H}_{ext.} = \sum_R p_R (\dim R)^{-2} \sum_{i,j} |R, i, j\rangle\langle R, j, i| \, , \qquad p_R = |\psi_R|^2$$

$$S_{EE} = -\sum_R p_R \log p_R + 2 \sum_R p_R \log \dim R \qquad \text{“log dim R edge mode”}.$$

Comment 1. Generalizations

- On a $d > 2$-dim. lattice, if we apply this definition across each boundary link,

$$S_{EE} = \text{Shannon edge term} + \log \dim R \text{ edge term} + \text{interior EE}. \quad (*)$$

- There's also a way to make sense of the extended Hilbert space in a continuum gauge theory, basically by extending the configuration space to include a gauge-group-valued degree of freedom along the boundary, lifting the boundary gauge symmetry to a global one. But I won't get into this today.

Comment 2. Connection with emergent gauge theory

- If we replace $\mathcal{H}_{ext.} \to \mathcal{H}_{UV}$ in an emergent gauge theory,

  $$S_{EE} = \text{Shannon edge term} + \text{log dim R edge term} + \text{interior EE}. \quad (*)$$

  holds (up to a state-independent constant). This explains the formula for EE in an emergent gauge theory that I showed you a few slides ago.

  An (artificial) example where this is obvious is if we take the UV theory to be lattice gauge theory without imposing the Gauss law at the vertices...

  More generally, the point is that Wilson loops factorize by definition.

Comment 3. Interpretation of the $\log d_R$ term

- Earlier, we saw an algebraic definition of EE. One can show that

$$S_{EE}^{\mathcal{H}_{ext.}}(\rho_A) = S_{alg,ginv}(A) + \text{log dim R edge.}$$

- Conversely, from a "totally IR" point of view, $\exists$ a center operator $\mathcal{L}_A$ s.t.

$$\langle R|\mathcal{L}_A|R\rangle = \log \dim R\,.$$

But $\mathcal{L}_A$ is a complicated, group-dependent function of the Casimirs (e.g. $\log\sqrt{4E^aE^a+1}$ for $G = SU(2)$). The completely obscures the canonical counting interpretation!

To summarize:

In a UV-finite theory with emergent extended objects (Wilson loops), the UV-exact EE of a region can be written in a "more IR" way, as an EE assigned to the extended objects contained within each region, plus a boundary term counting UV DOF's made visible when the extended objects are cut by the entangling surface.

Apply this formula to AdS/CFT

If one thinks of AdS/CFT as an emergent gauge theory, with the bulk emerging from the CFT, the area term looks a lot like the "$\log \dim R$" boundary term in the more IR way of writing the EE.

Ent. wedge reconstruction $\leftrightarrow$ **RT + FLM**

Region A in CFT $\sim$
$\mathcal{E}_A$ in bulk EFT

$$S_{EE}^{CFT}(A) = S_{alg,ginv}(\mathcal{E}_A) + \langle \text{center operator} \rangle \sim \frac{A}{4G_N}$$

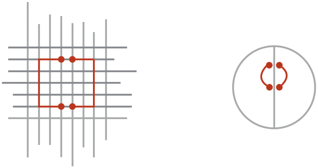
$\Downarrow$

$$S_{EE}^{CFT}(A) = S_{alg,ginv}(\mathcal{E}_A) + \log \dim R$$

If one assumes that the gauge theory formula can be used on the LHS, "$A/4G_N$" is a $\log \dim R$ term. This is the main point of this talk.

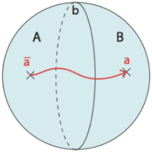
Evidence. The EE of a single string in AdS

Let's extend this conjecture to pert. string theory. Comparing a Wilson loop in an emergent gauge theory to a closed string in the bulk,



$$\begin{aligned} \frac{A}{4G_N} &= \text{log(boundary states)} \\ &= \text{\# ways to ``glue'' two open strings} \\ \frac{1}{4G_N} \sim \mathcal{O}(N^2) &= (\text{\#CP factors})^2? \end{aligned}$$

This picture is not new. It was suggested 20 years ago by Susskind and Uglum who studied which types of string diagrams would contribute to the black hole entropy in Euclidean thermodynamics.



- We can put a single string in AdS by putting a pair of heavy quarks in the CFT. The added EE on the boundary from the $q\bar{q}$ pair is [Lewkowycz, Maldacena]

$$S_{EE}(q,\bar{q}) \sim \log N + \dots$$

In the bulk, all we did was cut the single string with an entangling surface.

How can we test this?

Ideally we'd like to directly compute the "$\log d_R$" term of gravity.

The difficulty is that in the lattice gauge theory, this term came from a very well-defined operation which is not obvious how to generalize. First we added surface charges in all representations at the boundary links in the phase space of the lattice gauge theory, then we quantized as usual. We both need to understand what the analogous operation is in the continuum classical phase space of gravity, and then quantize it...

Two approaches...

Work in progress

- **Holography** gives us a boundary definition of string theory. If we also understand how the boundary realizes bulk locality, we can compute the bulk edge term from the boundary. Work underway to study this in the $c = 1$ matrix model.

- **Low-dimensional gravity.**
  - **3d QG** on AdS_3 is a Chern-Simons theory. The continuum edge modes of CS theories are well understood. Can this edge term be identified with the area?
  - What is the classical phase space analog of the extended Hilbert space construction in **2d QG**?

Summary

The conjecture "$\frac{A}{4G_N} = \log d_R$ term in the EE of gravity" seems well-motivated from AdS/CFT and other considerations.

It would be very nice to prove it explicitly and understand what RT/B-H entropy is counting.

Thank you!