# Phases of Three Dimensional Gauge Theories

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## Introduction

- In this talk we tackle the following question:

  what is the infrared behaviour of QCD-like theories in 2+1 dimensions?

- Outline our proposed answer to this question

  $\Longrightarrow$ Phase diagram of QCD-like theories

As a side bonus of this analysis this will lead to:

- Dualities between non-supersymmetric gauge theories!
- Intricate interplay of subtle anomalies
  - T-reversal symmetry
  - One-form symmetry

## Motivation

- Understanding the strong dynamics of gauge theories is interesting!
- $\mathrm{QCD}_{2+1}$ as domain walls/defects in $\mathrm{QCD}_{3+1}$
- Finite temperature $\mathrm{QCD}_{3+1}$
- Microscopic realization of nonabelian anyons (FQHE)
- $\mathrm{QCD}_{2+1}$ theories appear in the boundary of "topological insulators"

Phases of matter protected by T-reversal symmetry

## $\mathrm{QCD}_{2+1}$

- Choice of gauge group $G$

$$\mathcal{L}_{YM} = \frac{1}{2g^2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$$

- Choice of representation $R$ of $G$ for the fermions $\lambda$

$$\mathcal{L}_{\psi} = i\bar{\lambda}\not{D}\lambda$$

- Additional coupling in $2+1$ dimensions: Chern-Simons level $k$

$$\mathcal{L}_{CS} = \frac{k}{4\pi}\mathrm{Tr}\left(A \wedge dA + \frac{2}{3}A^3\right)$$

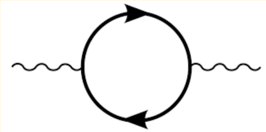
- Depending on $G$ there can additional topological invariants

$$\text{label theory as} \Longrightarrow \; G_k + \lambda_R$$

## Semiclassical Phases

Goal: determine the infrared dynamics of $G_k + \lambda_R$

- Identify semiclassical limits and analyze them
  - Give fermion a mass $m \in \mathbb{R}$: $\mathcal{L}_m = m\,\mathrm{Tr}(\bar{\lambda}\lambda)$
    - When $|m| \gg g^2$ integrate out $\lambda$.



Shifts the Chern-Simons coupling

$$k \to k + \mathrm{sign}(m)\frac{T(R)}{2}$$

$T(R)$: index of representation

  - Large $k$ limit. Gauge fields have a mass $\sim g^2 k$ and can be integrated out

## Strong Coupling Phase

- What happens at small $m$ and small $k$? In particular at $m = 0$ and $k = 0$
- Are there phase transitions as $m$ is varied?
- Does the topology of the phase diagram change as $k$ is lowered?

Strategy

- Identify global symmetries and 't Hooft anomalies
  - Continous symmetries: flavour symmetry, supersymmetry,...
  - Discrete symmetries: $C$, T-reversal, one-form symmetry,...
- Impose nonperturvative constraints
  - 't Hooft anomaly matching. Anomalies are RG monotones UV$\rightarrow$ IR
  - Use exact dualities: level/rank duality between Chern-Simons TQFTs

# Adjoint QCD

- Adjoint QCD: $G_k$+ adjoint $\lambda$
- Symmetries:
  - For $m = -\frac{k}{4\pi}$ theory has $\mathcal{N} = 1$ supersymmetry

$$\mathcal{L}_{susy} = \frac{k}{4\pi} \mathrm{Tr} \left( A \wedge dA + \frac{2}{3} A^3 - \bar{\lambda}\lambda \right)$$

is supersymmetry sponatenously broken? Witten index $\Longrightarrow$

$$\begin{array}{ll} k \geq h/2 & \text{supersymmetry is unbroken} \\ k < h/2 & \text{supersymmetry is expected to be broken} \end{array}$$

$h$: dual Coxeter number of $G$

  - There is a one-form symmetry $C[G]$ (center of $G$)
  - For $k = m = 0$ the theory has a T-reversal symmetry
  - Symmetries have 't Hooft anomalies. Also a "gravitational anomaly"

## $SU(N)_k$ adjoint QCD

- The $|m| \gg g^2$ asymptotic regions described by (for all $k$):

$$m > 0 \qquad SU(N)_{k+\frac{N}{2}} \ \text{TQFT}$$

$$m < 0 \qquad SU(N)_{k-\frac{N}{2}} \ \text{TQFT}$$

  - These TQFTs have nontrivial anyons, braiding,.... Nontrivial IR theory
  - The two asymptotic TQFTs are different

$$SU(N)_{k+\frac{N}{2}} \neq SU(N)_{k-\frac{N}{2}}$$

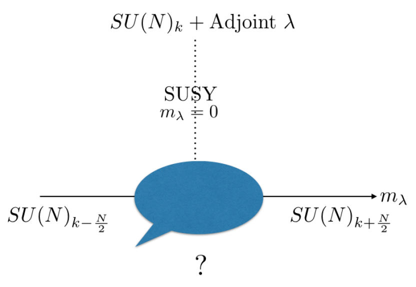
    $\implies$ there must be at least one phase transition as a function of $m$

  - IR TQFTs do have exact level/rank dual TQFTs

$$SU(N)_{k+\frac{N}{2}} \quad \longleftrightarrow \quad U\left(k+\frac{N}{2}\right)_{-N,-N}$$
$$SU(N)_{k-\frac{N}{2}} \quad \longleftrightarrow \quad U\left(k-\frac{N}{2}\right)_{-N,-N}$$

# Adjoint QCD for $k \geq N/2$

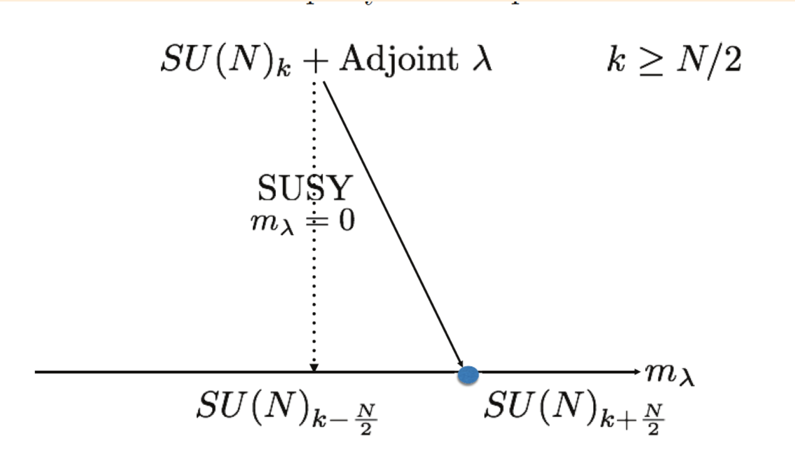
- What happens as $|m|$ is decreased? $m = -\frac{k}{4\pi} + m_\lambda$




- Obtain non-perturbative information at the $\mathcal{N} = 1$ supersymmetric point
  - For $k >> 1$ the fermion has a mass $m \sim -g^2 k$ and can be integrated out

    $\implies \quad SU(N)_{k-\frac{N}{2}}$ TQFT at the supersymmetric point
  - We propose that this IR TQFT is valid all the way down to $k = N/2$

# Phase Diagram of Adjoint QCD for $k \geq N/2$




- Two phases merge at a second order phase transition
- All anomalies automatically match

## Adjoint QCD for $k < N/2$

- The asymptotic large mass $SU(N)_{k\pm N/2}$ TQFTs phases still present
- We propose a new intermediate inherently quantum mechanical phase

$$\Longrightarrow \text{ intermediate TQFT}$$

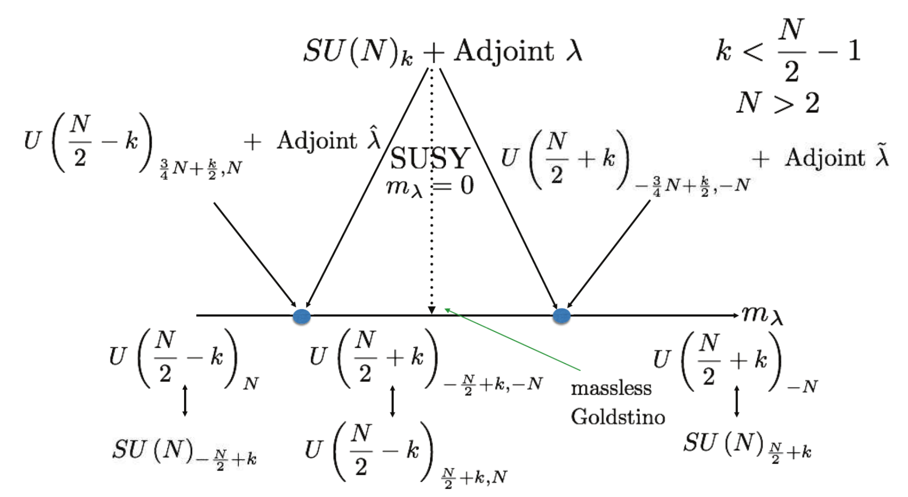
- There are two phase transitions for $k < \frac{N}{2}$
- Phase transitions have a dual gauge theory description

$$\Longrightarrow \text{ (nonsupersymmetric) gauge theory dualities!}$$

- At the supersymmetric point there is a massless Majorana particle

$$\text{supersymmetry breaking} \Longrightarrow \text{Goldstino}$$

## Phase Diagram of Adjoint QCD for $k < N/2$



- The anomalies match automatically (except T-reversal at $k = 0$)

- Consistent with level/rank duality of TQFTs

- New gauge theory dualities:

$$SU(N)_k + \text{adjoint } \lambda \quad \longleftrightarrow U\left(\tfrac{N}{2}+k\right)_{-\frac{3}{4}N+\frac{k}{2},-N} + \text{adjoint } \tilde{\lambda}$$

$$SU(N)_k + \text{adjoint } \lambda \quad \longleftrightarrow U\left(\tfrac{N}{2}-k\right)_{\frac{3}{4}N+\frac{k}{2},N} + \text{adjoint } \hat{\lambda}$$

- Generalized to other classical groups. New dualities:

$$SO(N)_k + \text{adjoint } \lambda \quad \longleftrightarrow SO\left(\tfrac{N-2}{2}+k\right)_{-\frac{3N}{4}+\frac{k}{2}+\frac{1}{2}} + \text{symmetric } \tilde{S}$$

$$SO(N)_k + \text{adjoint } \lambda \quad \longleftrightarrow SO\left(\tfrac{N-2}{2}-k\right)_{\frac{3N}{4}+\frac{k}{2}-\frac{1}{2}} + \text{symmetric } \hat{S}$$

$$Sp(N)_k + \text{adjoint } \lambda \quad \longleftrightarrow Sp\left(\tfrac{N+1}{2}+k\right)_{-\frac{3N}{4}+\frac{k}{2}-\frac{1}{4}} + \text{antisymmetric } \tilde{A}$$

$$Sp(N)_k + \text{adjoint } \lambda \quad \longleftrightarrow Sp\left(\tfrac{N+1}{2}-k\right)_{\frac{3N}{4}+\frac{k}{2}+\frac{1}{4}} + \text{antisymmetric } \hat{A}$$

- The adjoint QCD theory is $T$-reversal invariant

- But it has a non-trivial 't Hooft anomaly. On an unoriented manifold

$$T \cdot Z = e^{\frac{2\pi i \nu}{16}} Z$$

$\nu \in \mathbb{Z}_{16}$ is the T-reversal anomaly

- Intermediate is T-invariant

$$U(N/2)_{N/2,N} \leftrightarrow U(N/2)_{-N/2,-N}$$

with a 't Hooft anomaly

$$\nu_{TQFT} = \begin{cases} 2 & N = 2 \bmod 4. \\ -2, & N = 0 \bmod 4. \end{cases}$$

while

$$\nu_{UV} = \dim(SU(N)) \bmod 16$$

- T-reversal anomaly matches $\nu_{IR} = \nu_{UV}$ using that

$$\nu_{IR} = \nu_{TQFT} + \nu(\text{Goldstino}) = \nu_{TQFT} + 1$$

## Conclusions

- Explicit proposal for the dynamics of adjoint QCD in 2+1 d
- The infrared phases are nontrivial with phase transitions
- Consistency relies heavily on subtle 't Hooft anomalies
- Leads us to propose nonsupersymmetric gauge theory dualities
- Still leaning new facts about the dynamics of gauge theories!