Topological Defects in AdS

Some exact solutions from an extension of BPS theory

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Outline

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- Thus, our background is AdS_n with $n \geq 2$.

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- Finding one of these manifolds is analogous to picking a plane in $\mathbb{E}^n$.
- The appropriate notion of "flatness" is that Σ^q be a *totally geodesic* submanifold.

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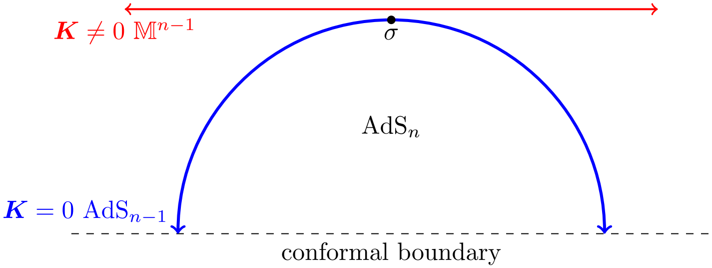

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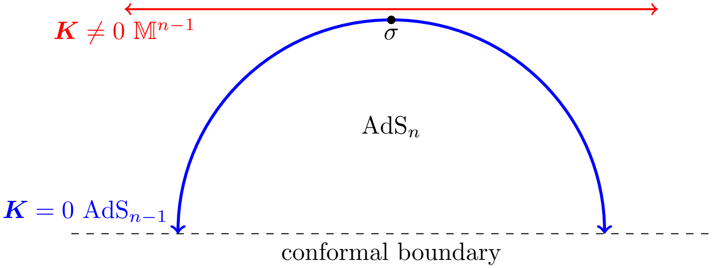

- In AdS_n, AdS_{n-1} is a totally geodesic submanifold; $\mathbb{M}^{n-1}$ is not.

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The Metric

- In the orthogonal coordinates $(\sigma, \boldsymbol{\nu}) = (\sigma^1, \ldots, \sigma^q, \nu^1, \ldots, \nu^l)$ that we employ, the metric of AdS_n is:

$$\mathrm{d}s^2_{\mathrm{AdS}_n} = \cosh^2\left(|k|^{1/2}\|\boldsymbol{\nu}\|\right)\,\mathrm{d}s^2_{\mathrm{AdS}_q} + \left[\mathrm{d}\nu^2 + \left(\frac{\sinh\left(|k|^{1/2}\|\boldsymbol{\nu}\|\right)}{|k|^{1/2}\|\boldsymbol{\nu}\|}\right)^2 \nu^2\,\mathrm{d}s^2_{S^{l-1}}\right]$$

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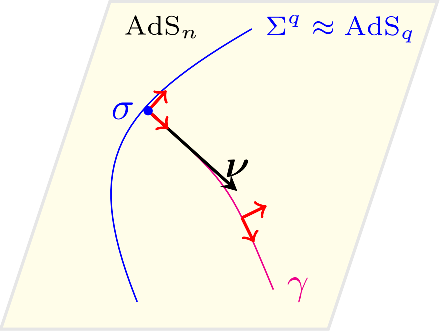

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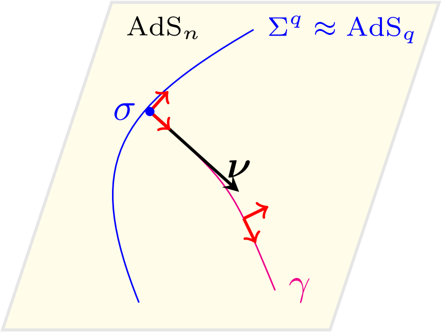

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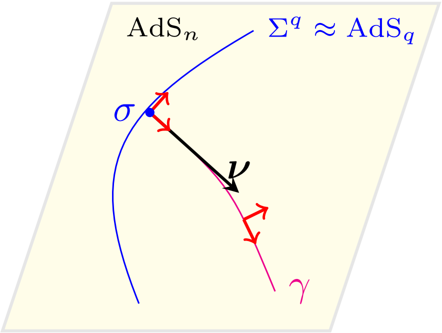

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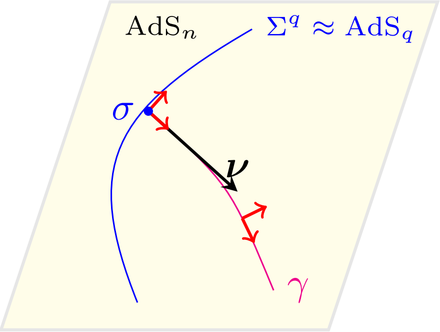

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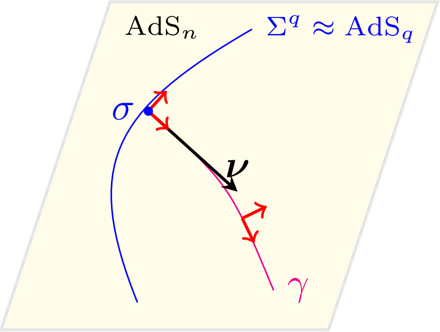

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- $\mathrm{d}s^2_{S^{l-1}}$ is the round metric for the $(l-1)$-sphere.

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Spherically-Symmetric Ansatz ($\mathrm{SO}(l)$ Symmetry)

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- In these coordinates, the action can be split into transverse and tangential components $I = -E_\perp \int_{\Sigma^q} \zeta_\Sigma$, where ζ_Σ is the volume element on the submanifold.

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- For convenience in computation, also define $h(\nu) = 1 - \frac{1}{2}\nu^2 f(\nu)$, where $\nu = \|\boldsymbol{\nu}\|$

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- With this ansatz, the transverse energy is:

$$E_\perp = V_{l-1} \int_0^\infty \mathrm{d}\nu \, [\cosh(\nu/\rho)]^q \; [\rho \sinh(\nu/\rho)]^{l-1}$$

$$1 \times \left\{ \left[\frac{1}{2} \phi'(\nu)^2 + \frac{(l-1)}{2} \frac{h(\nu)^2 \, \phi(\nu)^2}{[\rho \sinh(\nu/\rho)]^2} + U(\phi^2) \right] + \frac{(l-1)}{g^2} \left[\frac{1}{2} \frac{h'(\nu)^2}{[\rho \sinh(\nu/\rho)]^2} + \frac{(l-2)}{4} \frac{\left[h(\nu)^2 - 1\right]^2}{[\rho \sinh(\nu/\rho)]^4} \right] \right\}$$

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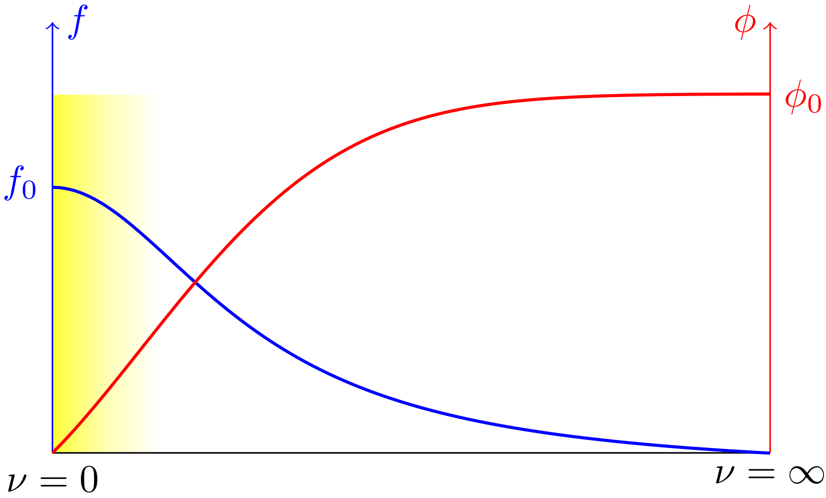
- The equations of motion are:

$$0 = -\frac{\mathrm{d}^2 \phi}{\mathrm{d}\nu^2} - \left(\frac{q}{\rho} \tanh(\nu/\rho) + \frac{(l-1)}{\rho} \coth(\nu/\rho) \right) \frac{\mathrm{d}\phi}{\mathrm{d}\nu} + \frac{\mathrm{d}U}{\mathrm{d}\phi} + \frac{(l-1)}{[\rho \sinh(\nu/\rho)]^2} \, h(\nu)^2 \, \phi(\nu)$$
$$0 = -\frac{\mathrm{d}^2 h}{\mathrm{d}\nu^2} - \left(\frac{q}{\rho} \tanh(\nu/\rho) + \frac{(l-3)}{\rho} \coth(\nu/\rho) \right) \frac{\mathrm{d}h}{\mathrm{d}\nu} + g^2 \phi(\nu)^2 \, h(\nu) + \frac{(l-2)}{[\rho \sinh(\nu/\rho)]^2} \left(h(\nu)^2 - 1 \right) h(\nu)$$

Asymptotics: Boundary conditions



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$$\phi(\nu) \xrightarrow{\nu\to\infty} \phi_{\text{far}}(\nu) = \phi_0 - \psi_* e^{-m_*\nu}, \qquad l \geq 1$$

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$$\phi(\nu) \xrightarrow{\nu\to 0} \phi_{\text{near}}(\nu) = \phi'(0)\nu + O\left(\nu^3\right), \qquad l \geq 1$$
$$h(\nu) \xrightarrow{\nu\to 0} h_{\text{near}}(\nu) = 1 - \frac{1}{2}f(0)\nu^2 + O\left(\nu^4\right), \qquad l \geq 2$$

- Far region: $\phi(\nu) \to \phi_0$, $h(\nu) \to 0$

$$\phi(\nu) \xrightarrow{\nu\to\infty} \phi_{\text{far}}(\nu) = \phi_0 - \psi_* e^{-m_*\nu}, \qquad l \geq 1$$
$$h(\nu) \xrightarrow{\nu\to\infty} h_{\text{far}}(\nu) = h_* e^{-\mu_*\nu}, \qquad l \geq 2$$

$$m_* = \frac{n-1}{2\rho} + \sqrt{m_\phi^2 + \left(\frac{n-1}{2\rho}\right)^2} \qquad \mu_* = \frac{n-3}{2\rho}\,, + \sqrt{\left(\frac{n-3}{2\rho}\right)^2 + m_A^2}$$

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Outline

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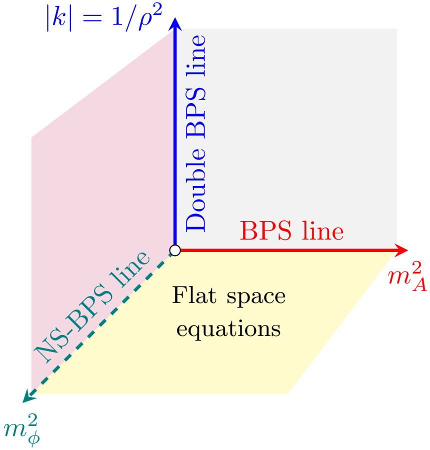
- BPS solutions are in the limit $m_\phi^2 \downarrow 0$, $1/\rho^2 \downarrow 0$.



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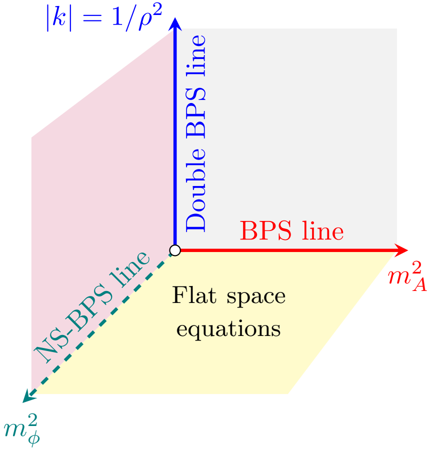

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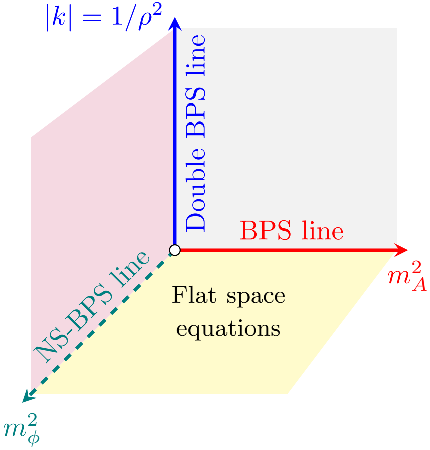

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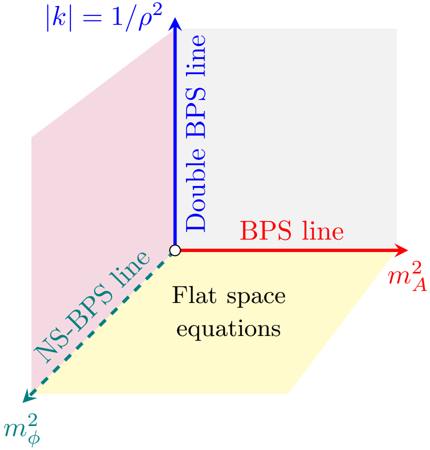
- In our model, the *curvature of the background* sets the length scale of the physics.



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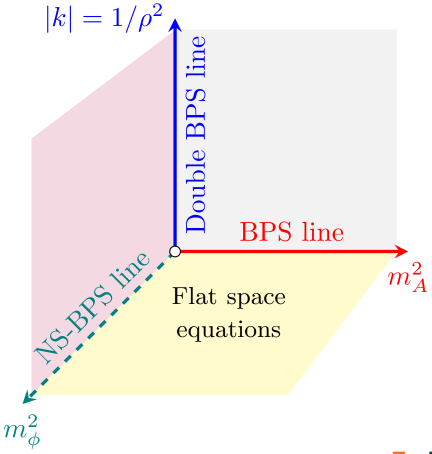

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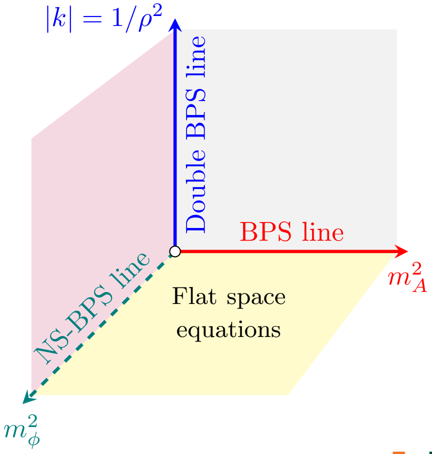

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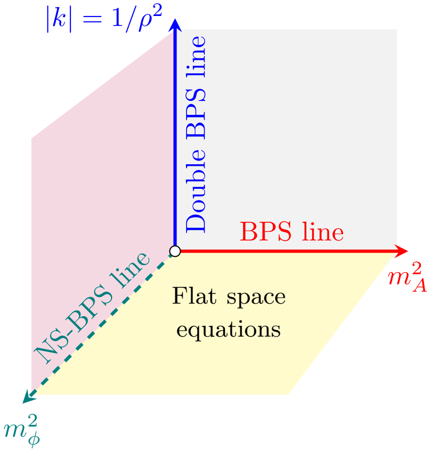

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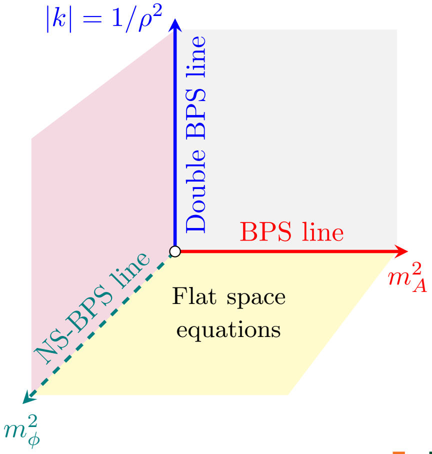
- We will use our subsequent solutions as a stepping-off point in a perturbative expansion solution of the full equations of motion.



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- Note that these are the only combinations that ever appear in our EOM.

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- If we can find h, then we can get a standalone equation for ϕ

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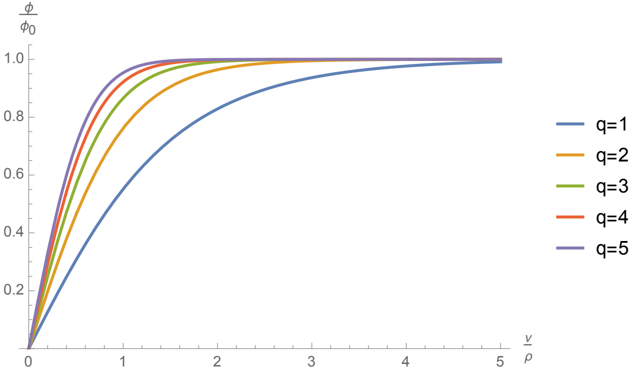

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$$\phi(\nu) = \mathcal{N}_{(3,2)}^{-1} \, 2\big(\tanh\nu \; J_0(\tanh\nu) - J_1(\tanh\nu)\big) = \mathcal{N}_{(3,2)}^{-1} \, 2\tanh\nu \; J_1'(\tanh\nu)$$

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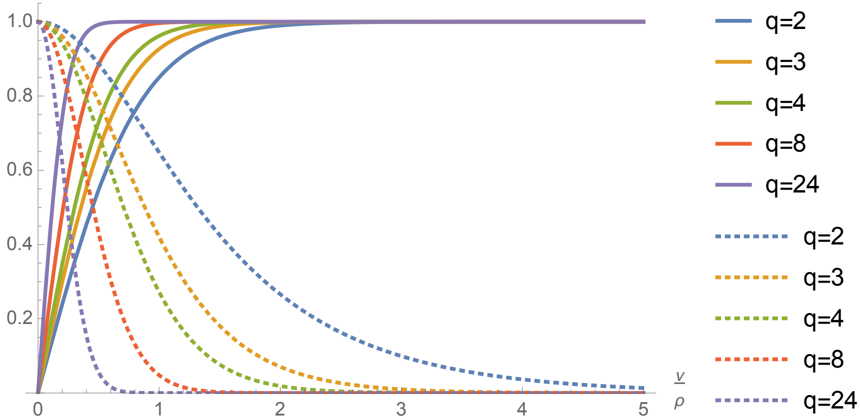

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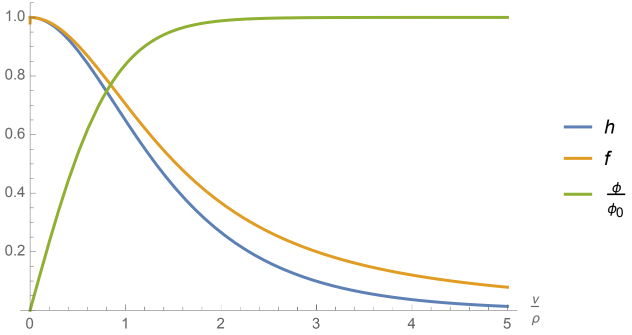
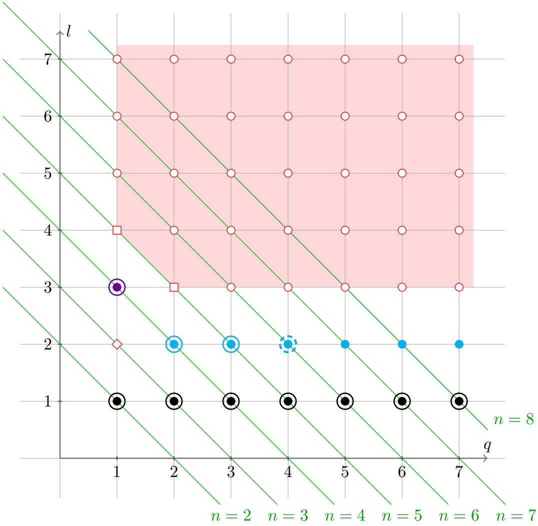


Other Solutions



- Concentric Circles: Exact double BPS solution.

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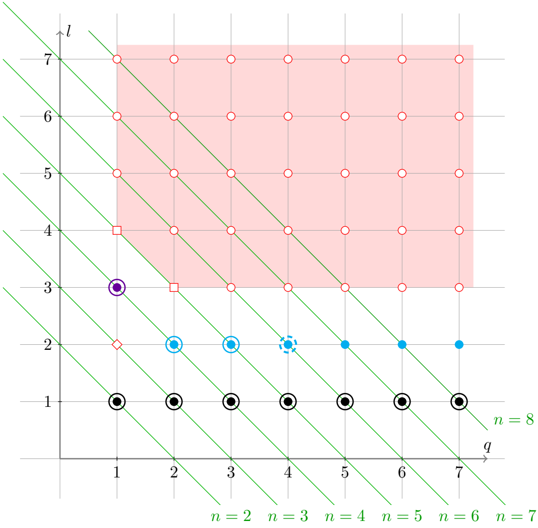

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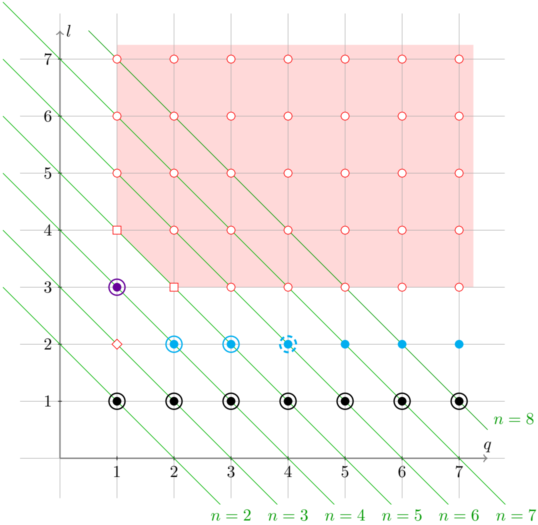

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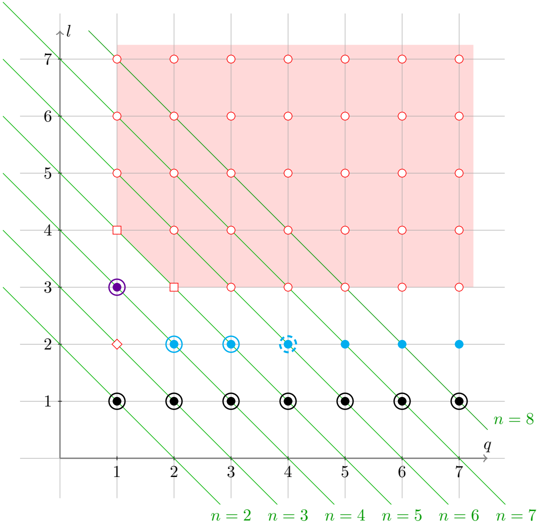

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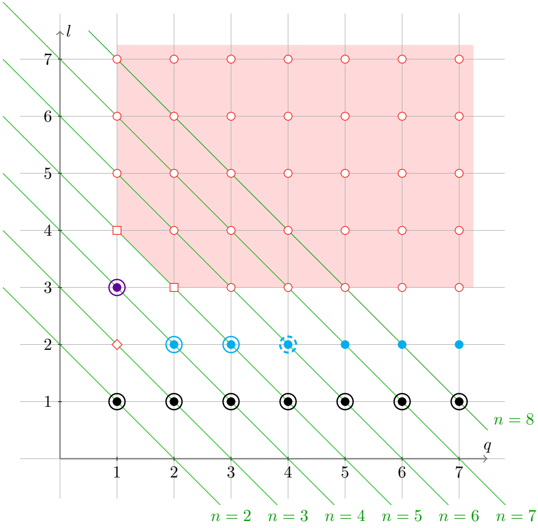

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- For non-exact solutions, we employ a numerical power-series technique.

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- This in turn can be used to find an expansion for the transverse energy $E_\perp$.
- In this expansion, the leading term goes like $\frac{1}{m_A^2}$, the subleading term goes like $(m_A^2)^0(m_\phi^2)^0$, and all subsequent terms go like $(m_A^2)^i(m_\phi^2)^j$, with $i, j \in \mathbb{Z}_+$.

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This work has been submitted for publication.

For a complete list of references, please see the manuscript on the arXiv: 1708.06327.

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