Quantization of CPT-Violating Negative-Energy Cherenkov states

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Talk presented at Miami 2017 - D C, P McDonald, J Noordmans, and R Potting PRD 2017
- D C, PLB 2017.

Overview of Talk

- Introduction
- Classical Mechanics lagrangians in the SME
- Extended Hamiltonian Formalism
- Quantization Procedure and Normalization Factors

Introduction

Theories involving negative-energy states are usually ruled out

Ex: Simple tachyonic neutrino model

$$L = i\overline{\psi}\gamma_5 \not{\partial}\psi - m_\nu \overline{\psi}\psi$$

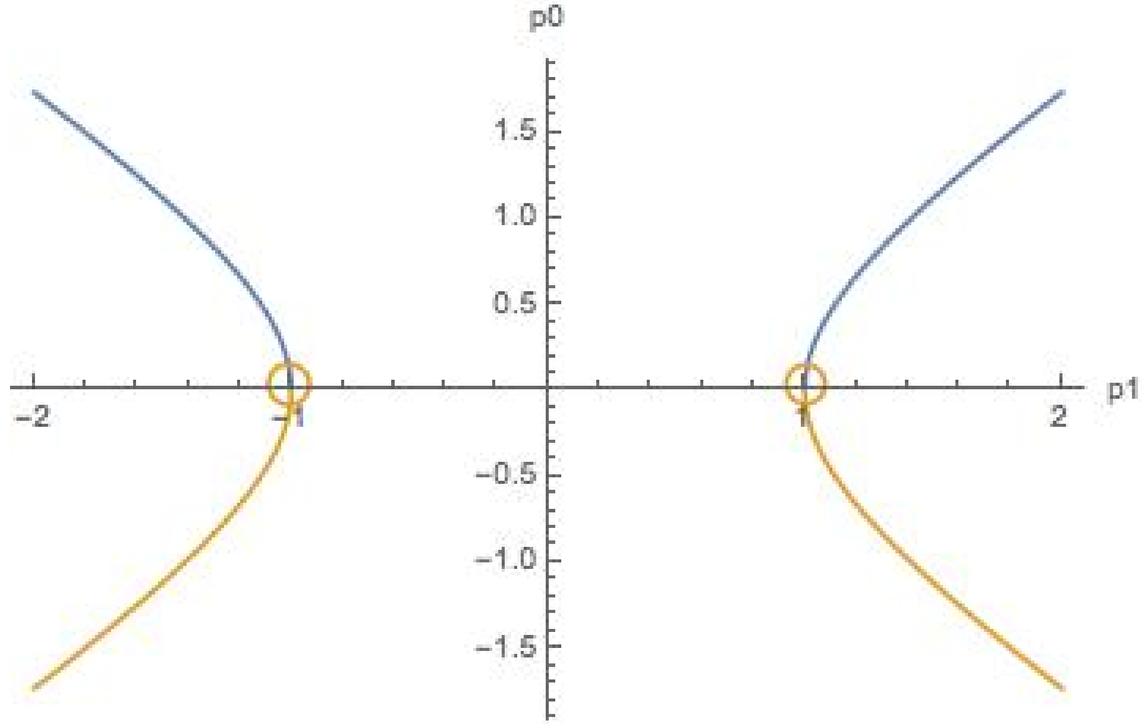
- A. Chodos, A. I. Hauser, and V. A. Kostelecky, PLB 1985

has dispersion relation

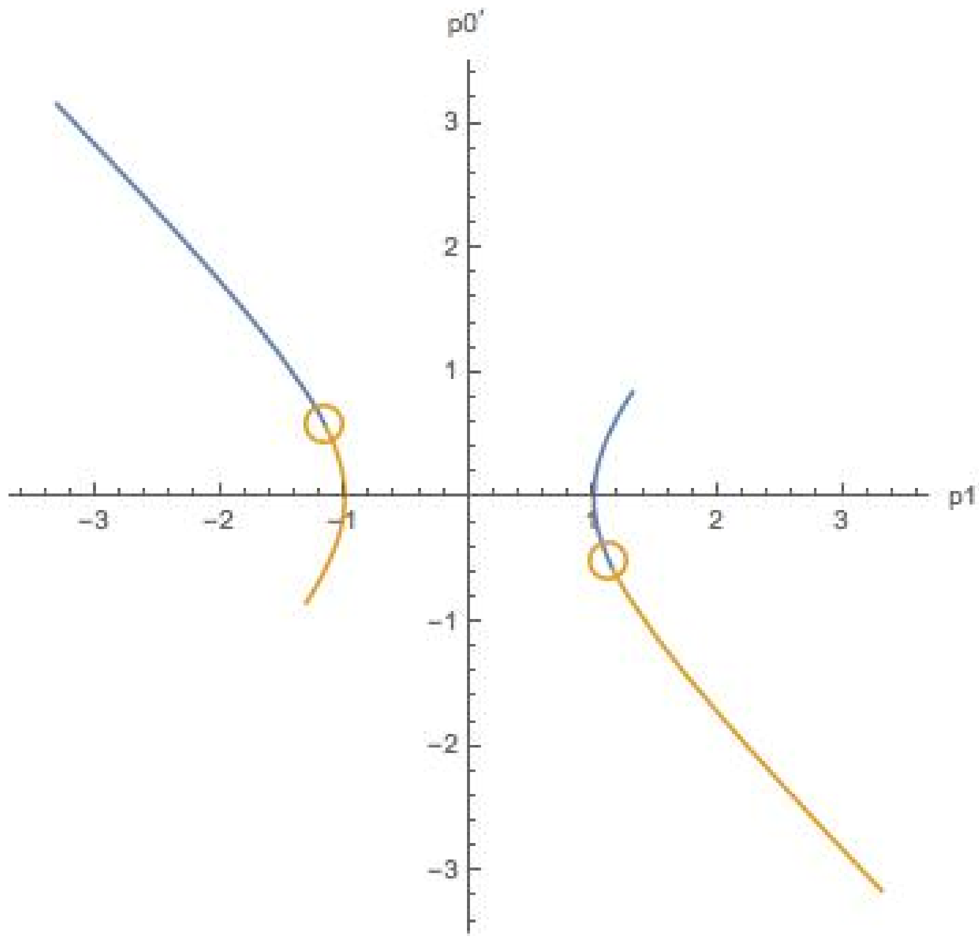
$$p^2 = -m_\nu^2$$

is an effective description at classical level, but has serious problems in QFT

Attempts at quantization in standard QFT lead to failure due to non-covariant nature of negative-energy state reinterpretation



Boost to moving frame moves "reinterpretation" points



Similar issues occur in Lorentz violating theories

For example, the perturbed CPT-violating, massive photon model

$$\mathcal{L}_A = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu}A^\lambda F^{\mu\nu} + \tfrac{1}{2}m_\gamma^2 A_\mu A^\mu$$

has perturbed dispersion relation factor

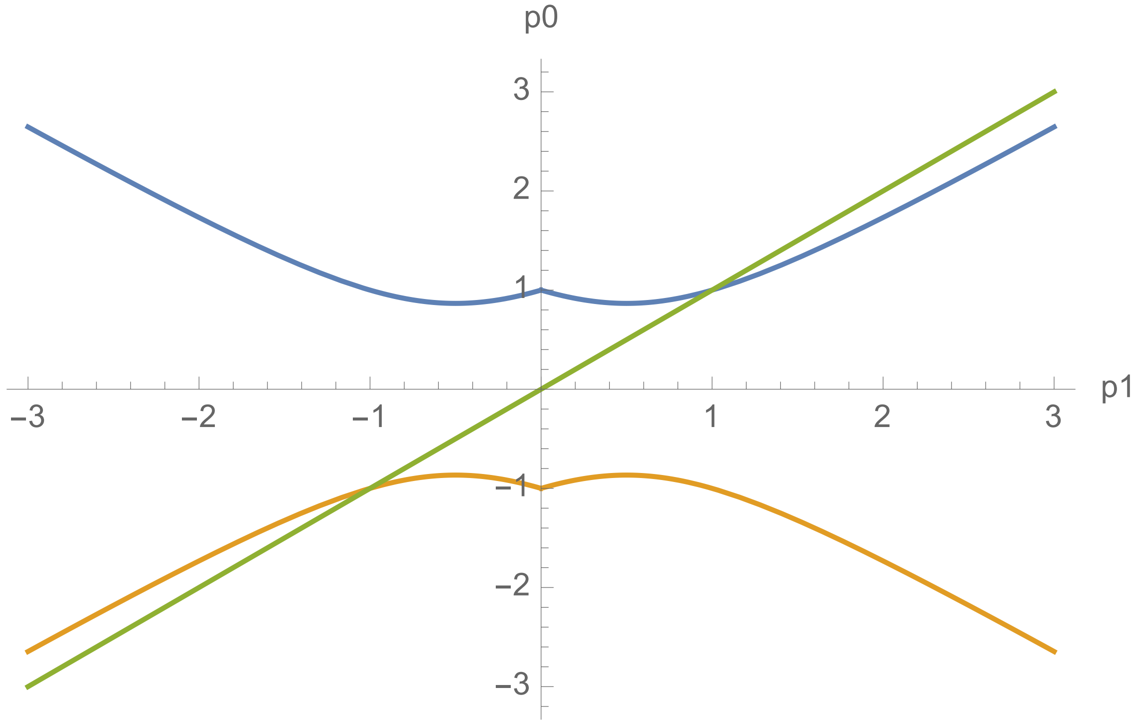
$$R_T(p) = \frac{1}{4}(p^2 - m_\gamma^2)^2 - (p \cdot k_{AF})^2 + p^2 k_{AF}^2 = 0$$

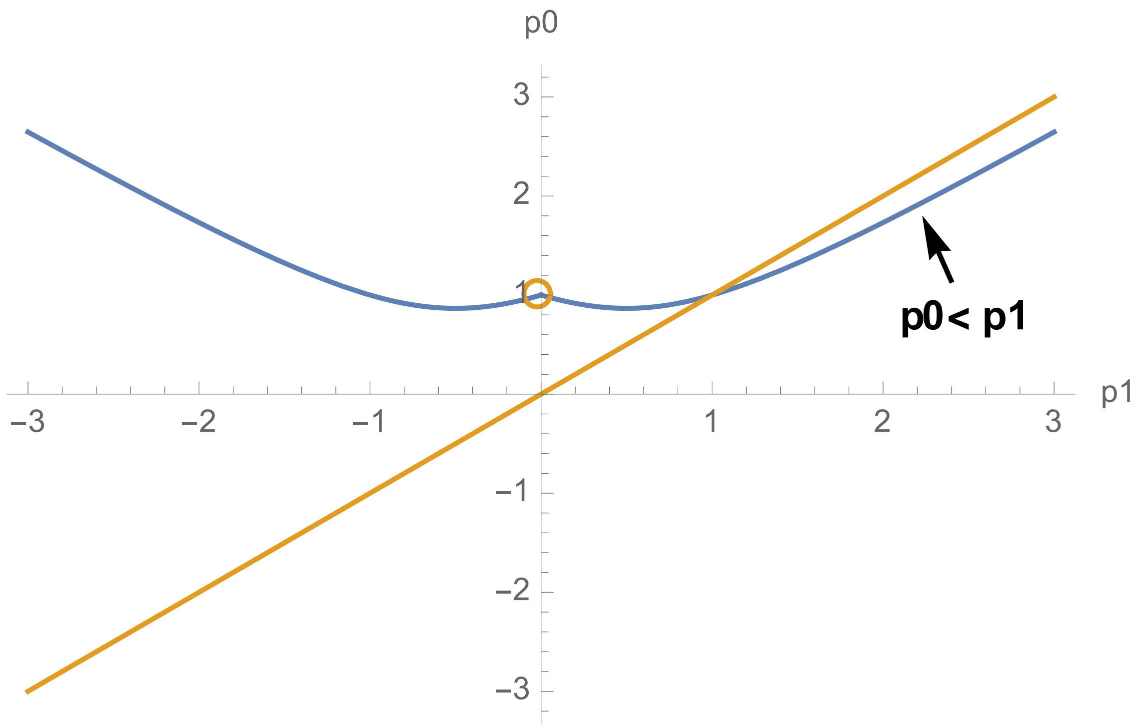
which has an observer covariant factorization $R_T(p) = R_+(p)R_-(p)$

$$R_\pm(p) = \frac{1}{2}(p^2 - m_\gamma^2) \pm \sqrt{(p \cdot k_{AF})^2 - p^2 k_{AF}^2}$$

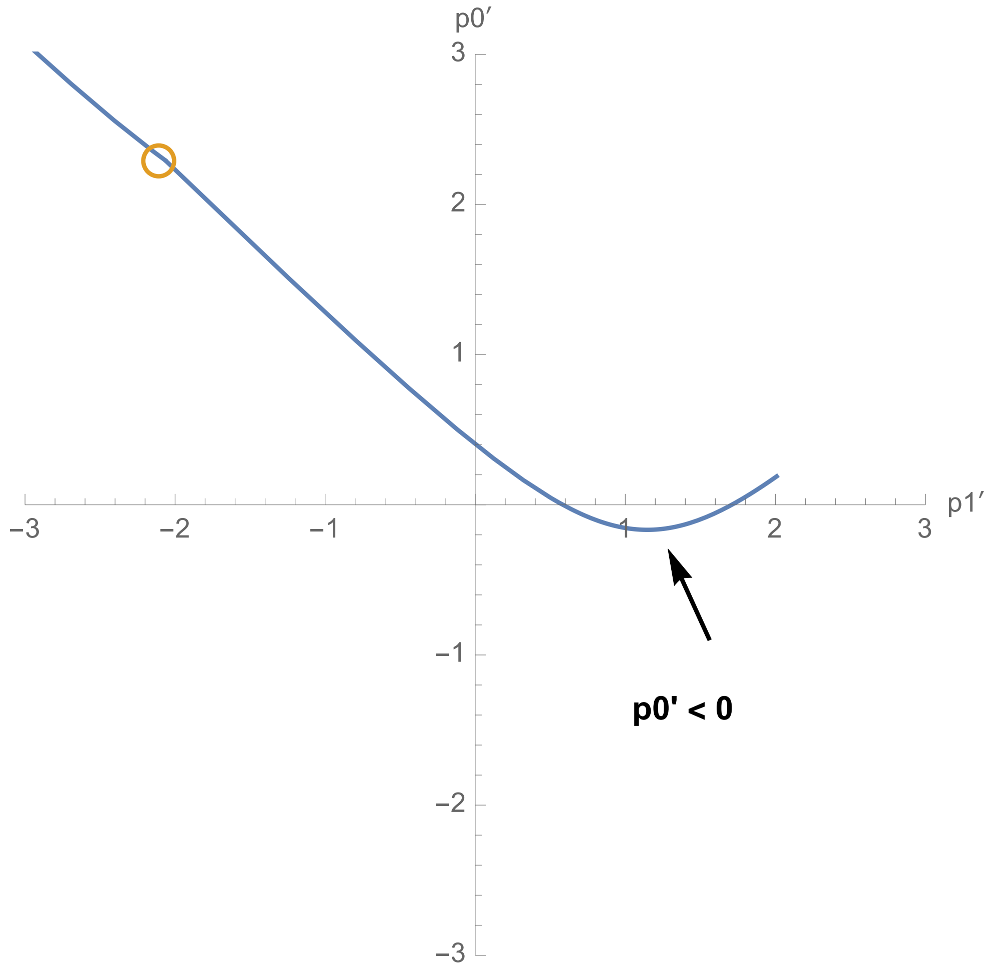
Have solutions with $p^2 < 0$ at high-momenta for $R_+ = 0$

These solutions separate into positive and negative-energy states





Can see region on $R_+(p) = 0$ where $p^2 < 0$ (exaggerated here...)



Boost can take space-like states into frame with $p_0' < 0$

Possible resolutions to this?

- Theory may only be valid in concordant frames where the LV parameters are small ($b < m$)

  At high energies, higher-order operators are expected to contribute and rescue the problem

  - A. Kostelecky and R. Lehnert PRD 2001

- Theory may in fact include these states, can interpret these as producing Cherenkov radiation

  But how to quantize? (naive re-interpretation fails...)

  $\rightarrow$ look at classical mechanics formulation

Classical Mechanics Lagrangians in the SME

Traditional method of computing classical Lagrangians

– Kostelecký and Russell, PLB 2010; Kostelecký PLB 2011.

- Example is dispersion relation for b^μ (fermion, isomorphic to photon case...)

$$\mathcal{R}(p) = \frac{1}{4}(p^2 - m^2 + b^2)^2 - (b \cdot p)^2 + m^2 b^2 = 0$$

- Three-velocity $\vec{v} = d\vec{x}/dt$ defined using implicit differentiation

$$v_i = \frac{\partial p_0}{\partial p^i}$$

- Lagrangian given by Legendre transform $L = \vec{u} \cdot \vec{p} - p^0(\vec{p})$

- expression for $\vec{v}(\vec{p})$ is inverted for $\vec{p}(\vec{v})$ to give

$$L_{\pm}[\vec{v}, x] = -m\sqrt{1 - \vec{v}^2} \mp \sqrt{(b^0 - \vec{b} \cdot \vec{v})^2 - b^2(1 - \vec{v}^2)}$$

  where the signs represent two valid solutions that reduce to the standard case when $b^\mu \to 0$.

- Above procedure is not explicitly covariant, so can introduce $u^0(\lambda)$ function and an arbitrary path parametrization $\lambda(t)$ so that the action appears covariant in terms of $u^\mu = dx^\mu/d\lambda$

$$L_{\pm}[u^\mu, x] = -m\sqrt{u^2} \mp \sqrt{(b \cdot u)^2 - b^2 u^2}$$

Some issues with above approach

- u^μ has a "gauge" degree of freedom, not fully determined by equation of motion

- momentum $p^\mu = -\frac{\partial L}{\partial u_\mu}$ gives

$$p^\mu = \frac{m u^\mu}{\sqrt{u^2}} \pm \frac{(u \cdot b) b^\mu - b^2 u^\mu}{\sqrt{(b \cdot u)^2 - b^2 u^2}}$$

  satisfies $p(\xi u) = p(u)$, which means it is not invertible without imposing a constraint on u^μ (eg. $u^2 = 1$ for proper time)

- singular points exist where p^μ is indeterminate

- Calculation of the relativistic hamiltonian yields zero

$$\mathcal{H} = p^0 u^0 - \vec{p} \cdot \vec{u} - L = 0$$

(since $L = -u_\mu p^\mu$) yielding no useful hamiltonian formulation

- Unclear how L_+ and L_- relate to solutions of

$$\mathcal{R}(p) = \frac{1}{4}(p^2 - m^2 + b^2)^2 - (b \cdot p)^2 + m^2 b^2 = 0$$

Extended Hamiltonian Formalism (Dirac)

Can introduce new variable $e(\lambda)$ as lagrange multiplier

$$S^*_\pm = -\int \left[m e^{-1} u^2 \pm \sqrt{(b \cdot u)^2 - b^2 u^2} - \frac{e}{m} \mathcal{R}_\mp(p, x) \right] d\lambda,$$

With the observer covariant factorization

$$\mathcal{R}(p) = \mathcal{R}_+(p) \mathcal{R}_-(p)$$

and

$$\mathcal{R}_\pm = \frac{1}{2} \left(p^2 - m^2 - b^2 \right) \pm \sqrt{(b \cdot p)^2 - b^2 p^2}$$

agrees with the previous action "on-shell" where $\mathcal{R}_\pm = 0$, but modified action applies for unconstrained variations of u^μ and p^μ

Factorization arises naturally when considering the Dirac equation '*off-shell*' spinors

$$\frac{1}{2}(\not{p} - m - b_\mu \gamma_5 \gamma^\mu) u_\pm(p) = \mathcal{R}_\pm(p) u_\pm(p)$$

with $\mathcal{R}_\pm(p) = \frac{1}{2}(p^2 - m^2 - b^2) \pm \sqrt{(b \cdot p)^2 - b^2 p^2}$ as above

acting on spinors $u_\pm = (\not{p} + m - \gamma_5 \not{b}) w_\pm$ gives condition

$$\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\mu\nu} p^\alpha b^\beta w_\pm = \pm\sqrt{(b \cdot p)^2 - b^2 p^2} w_\pm$$

$\rightarrow \mathcal{R}_\pm$ correspond to these eigenstates

Writing action in the form

$$S^* = \int \left[-p^\mu u_\mu - \mathcal{H}^*\right] d\lambda = \int L^* d\lambda$$

allows for identification of extended relativistic hamiltonians

$$\mathcal{H}^*_\pm = -\frac{e}{m}\mathcal{R}_\mp(p,x) = -\frac{e}{2m}\left(p^2 - m^2 - b^2 \pm 2\sqrt{(b\cdot p)^2 - b^2p^2}\right)$$

and hamilton's equations give (valid since all p^μ independent...)

$$u^\mu = -\frac{\partial \mathcal{H}^*}{\partial p_\mu} = \frac{e}{m}\left(p^\mu \mp \frac{(b\cdot p)b^\mu - b^2p^\mu}{\sqrt{(b\cdot p)^2 - b^2p^2}}\right)$$

which can be inverted to give

$$p^\mu = \frac{mu^\mu}{e} \pm \frac{(u\cdot b)b^\mu - b^2u^\mu}{\sqrt{(b\cdot u)^2 - b^2u^2}}$$

Expression of $\mathcal{H}^*$ in terms of u^μ gives simple form

$$\mathcal{H}^*_\pm = -\frac{m}{2e}(u^2 - e^2)$$

Plugging this into the action gives the extended lagrangian

$$L^*_\pm[u^\mu, x, e] = -\frac{m}{2e}u^2 \mp \sqrt{(b \cdot u)^2 - b^2 u^2} - \frac{em}{2}$$

Setting $\partial L^*/\partial e = 0$ gives $e = \sqrt{u^2}$ ('einbein') and L^* reduces to

$$L_\pm[u^\mu, x] = -m\sqrt{u^2} \mp \sqrt{(b \cdot u)^2 - b^2 u^2}$$

and sends $\mathcal{H}^* \to 0$, hence it is useful to retain e as an auxiliary variable so that $\mathcal{H}^* \neq 0$ can be used

Derivation of Hamilton's equation works as p^μ unconstrained

$$L^*[u^\mu, x^\mu, e] = -p \cdot u - \mathcal{H}^*[p^\mu, x^\mu, e]$$

Taking differentials of both sides and using the E-L Equations together with $\partial L^*/\partial u^\mu = -p^\mu$ gives Hamilton's equations

$$\frac{\partial \mathcal{H}^*}{\partial p^\mu} = -u^\mu, \quad \dot{p}_\mu = \frac{\partial \mathcal{H}^*}{\partial x^\mu}, \quad \mathcal{H}^* = 0$$

where the derivatives are now all taken 'off-shell' maintaining explicitly covariant form throughout the Legendre Transformation

Reproduces geodesic equation when $p(u)$ and $u^\mu = dx^\mu/d\lambda$ used

Hamilton's equations give classical dynamics as

$$\frac{\partial \mathcal{H}^*}{\partial x_\mu} = \dot{p}^\mu, \quad \text{with constraint} \quad \mathcal{H}^* = 0$$

which is the on-shell condition.

In addition, $\mathcal{H}^* = -\frac{m}{2e}(u^2 - e^2) = 0 \rightarrow u^2 = e^2 = 1$ for proper time parametrization, so group velocity always < 1

- D. C., PLB 2017.

Quantization

Typical field expansion in the conventional case contains observer covariant phase space factor

$$\int \frac{d^3\vec{p}}{2p^0(\vec{p})} = \int d^4p \; \delta(p^2 - m^2)\Theta(p^0)$$

is problematic in frames where $p^0 \leq 0$ where commutators

$$[a(p), a^\dagger(p)] = (2\pi)^3 2p^0(\vec{p})\delta(\vec{p} - \vec{p}')$$

vanish or go negative (leads to negative-norm states)

(wrong approach with CPT-violation since satisfies $p^2 - m^2 \neq 0$)

Fix: Use the correct relativistic hamiltonian for the theory

$$\mathcal{H}^*_\pm = -\frac{e}{2m}\left(p^2 - m^2 - b^2 \pm 2\sqrt{(b\cdot p)^2 - b^2p^2}\right)$$

$$\int d^4p\ \frac{-e}{2m}\delta(\mathcal{H}^*_\pm(p)) = \int \frac{d^3\vec{p}}{\Lambda^{0\prime}_\pm(p)}$$

with

$$\Lambda^{0\prime}_\pm(p) = -\left(\frac{2m}{e}\right)\frac{\partial\mathcal{H}^*_\pm}{\partial p^0} = 2\left(p^0 \pm \frac{(b\cdot p)b^0 - b^2p^0}{\sqrt{(b\cdot p)^2 - b^2p^2}}\right)$$

enforces the correct dynamics on surface $\mathcal{H}^*_\pm = 0$ and

Since $u^2 = u_0^2 - \vec{u}^2 = 1$ (in proper time parameterization) have $u^0 = -\partial\mathcal{H}^*_\pm/\partial p^0 > 0$ in all frames so denominator is positive

Using this factor, field expansions like

$$A^\mu(x) = \int \sum_{\pm} \frac{d^3\vec{p}}{\Lambda^{0\prime}_{\pm}(p)} \epsilon^\mu a^\dagger(p) e^{-ip\cdot x} + c.c.$$

and corresponding phase space factors are well-defined in all frames as the denominator > 0

$$[a(\vec{p}), a^\dagger(\vec{p}')] = (2\pi)^3 \Lambda^{0\prime}_{\pm}(p)\delta(\vec{p} - \vec{p}')$$

is also well-defined in all frames

- D.C., P. McDonald, J. Noordmans and R. Potting PRD 2017

Define canonical momenta as $\pi^\mu = \partial\mathcal{L}/\partial\dot{A}_\mu$

$$\pi^\mu(x) = F^{\mu 0}(x) + \epsilon^{0\mu\alpha\beta}(k_{AF})_\alpha A_\beta(x) - \eta^{\mu 0}\tfrac{1}{\xi}\partial_\nu A^\nu(x)$$

Quantization rules are implemented as

$$[A_\mu(t,\vec{x}),\pi^\nu(t,\vec{y})] = i\delta^\nu_\mu\delta^3(\vec{x}-\vec{y}) \ , \quad [A_\mu(t,\vec{x}),A_\nu(t,\vec{y})] = 0$$

with field expansion

$$A^\mu(x) = \int\sum_{\pm}\frac{d^3\vec{p}}{\Lambda^{0\prime}_{\pm}(p)}\epsilon^\mu a^\dagger(p)e^{-ip\cdot x} + c.c.$$

impies

$$[a(\vec{p}),a^\dagger(\vec{p}\,')] = (2\pi)^3\Lambda^{0\prime}_{\pm}(p)\delta(\vec{p}-\vec{p}\,')$$

with $\Lambda^{0\prime}_{\pm}(p) > 0$ in *all* observer frames

Summary

- Space-like states can be handled consistently in classical mechanics limit using the extended hamiltonian formalism

- Quantization can be successfully extended to non-concordant frames using the classical dynamics to normalize fields

- Phase space factors also use the same method leading to consistent calculation of Cherenkov processes

- CPT-violating theories that involve negative energy states are not automatically ruled out as was previously postulated