Anisotropic EoS and stellar structure equations for magnetized compact stars

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Outline

Introduction / Motivation

1. EoS dependent on the magnetic field: anisotropies.
   - Magnetized Strange Stars / Magnetized White Dwarfs

2. Static equilibrium structure equations: TOV equations
   - Magnetized Strange Stars MSSs
   - Magnetized White Dwarfs MWDs

3. Static anisotropic structure equilibrium equation: cylindrical metric
   - Magnetized Strange Stars MSSs
   - Magnetized White Dwarfs MWDs

4. Conclusions and perspective of the work
Magnetic field is present in all scale of the Universe

In Astrophysics there are Magnetars with $10^{15} \text{ G}$ at the surface inside B goes up $10^{18} \text{ G}$

Early Universe huge magnetic field $10^{24} \text{ G}$ would explain intergalactic field $10^{-6} \text{ G}$

It is an open problem the origin of these field but they play a profound role in the stellar/cosmic physics

Heavy ions colliders, strong magnetic field at least locally up to $m_{\pi}^2 \sim 10^{18} \text{ G}$
Introduction/Motivation

....focus in Astrophysical scenarios: Compact objects...

Looking for more realist models of CO which could explain some observables:

--- maximum values of Masses of NS/SSs

- PSR J1614-2230, Demorest et al Nature, 467, 1081 2010

- PSR J0348+04232 J Antoniadis et al Science, 2013,


--- upper limits for the values of the magnetic field in CO

\[ M_{\text{max}} = 1.97 M_\odot \]

\[ M_{\text{max}} = 2.01 M_\odot \]

\[ M_{\text{max}} \gtrsim 1.44 M_\odot \]
To do these tasks:

EoS in presence of magnetic field

"MICROSCOPIC PHYSICS"

but also to explore

structure stellar equilibrium equations and the

"MACROSCOPIC CONSEQUENCES"
We have been devoted to study “microscopic physics”


- Magnetized Quark Stars: MSQM and MCFL.

- EoS the inclusion of AMM moment, and the insignificant effect!
Anisotropic pressures due to the presence of constant \( B \) in \( z \)-direction\(^1\)

\[
T^a_b = \begin{pmatrix}
P_\perp & 0 & 0 & 0 \\
0 & P_\perp & 0 & 0 \\
0 & 0 & P_\parallel & 0 \\
0 & 0 & 0 & \epsilon
\end{pmatrix}
\]

\[
P_\parallel = -\Omega^B f - \frac{B^2}{8\pi} \quad P_\perp = -\Omega^B f - M_f B + \frac{B^2}{8\pi}
\]

More realistic EoS to take into account the magnetic field \( B \)

\[
P \rightarrow f(\epsilon(\mu,T,B))
\]


Anisotropic EoS/matter

we start from the thermodynamical potential for charge fermion system

\[ \Omega_f(\mu_f, B, T) = -\frac{d_f e_f B}{\beta} \sum_l \sum_{p_4} \int_{-\infty}^{\infty} \frac{dp_3}{(2\pi)^2} \ln \det G_f^{-1}(\vec{p}^*) \]

\[ \vec{p}^* = (ip^4 - \mu_f, 0, \sqrt{2e_f Bl}, p^3) \quad l = 0, 1, 2, \ldots \]

\[ G_f^{-1} = \det[\vec{p}^* \cdot \gamma - m_f] \]

\[ \epsilon_f = \sqrt{p_3^2 + m_f^2 + 2|e_f B|l} \]

\[ \Omega_f = \Omega_f^{\text{vac}}(B, 0, 0) + \Omega_f^{\text{est}}(B, \mu, T) \]
Anisotropic EoS/matter

In astrophysical scenario $T \ll T_F$, so the degenerate limit is considered ($T=0$)

\[
\Omega_{\text{vac}}^f (B, 0, 0) = - \frac{e_f B}{4\pi^2} \sum_{l=0}^{\infty} \int dp_3 \left| \varepsilon_f \right| = - \frac{\alpha B^2}{6\pi} \ln \left( \frac{B}{B_c} \right)
\]

\[
\Omega_{\text{st}}^f (B, \mu_f, 0) = - \frac{d_f e_f B}{4\pi^2} \left( \sum_{l=0}^{l_{\max}} \alpha_l \left( \mu_f p_f^F - \varepsilon_{0f}^2 \ln \frac{\mu_f + p_f^F}{\varepsilon_{0f}} \right) \right)
\]

\[
\Omega_f = \Omega_{\text{vac}}^f (B, 0, 0) + \Omega_{\text{st}}^f (B, \mu_f, 0)
\]

\[
p_f^F = \sqrt{\mu^2 - m^2 - 2eBl}
\]

\[
\varepsilon_f = \sqrt{p_3^2 + m^2 + 2eBl}
\]

\[
\varepsilon_0 = \sqrt{m^2 + 2eBl}
\]

\[
\alpha_l = 2 - \delta_{l0}
\]
MSQM in Bag Model
EoS+stellar chemical equilibrium conditions

\[ \varepsilon = \sum_f (\Omega^B_f + \mu_f N_f) + \frac{B^2}{8\pi} + B_{\text{bag}}, \]

\[ P_\parallel = -\sum_f \Omega^B_f - \frac{B^2}{8\pi} - B_{\text{bag}}, \]

\[ P_\perp = -\sum_f (\Omega^B_f - BM_f) + \frac{B^2}{8\pi} - B_{\text{bag}}, \]

\[ \mu_d = \mu_s \]
\[ \mu_u + \mu_e - \mu_d = 0 \]
\[ 2N_u - N_d - N_s - 3N_e = 0 \]
\[ N_u + N_d + N_s - 3n_B = 0. \]

\[ N = -\frac{\partial \Omega_f}{\partial \mu} \]
\[ M = -\frac{\partial \Omega_f}{\partial B} \]
EoS+stellar chemical equilibrium conditions MWD

\[ \varepsilon = \Omega_e + \mu_e N_e + m_N \frac{A}{Z} N_e + \frac{B^2}{8\pi}, \]

\[ P_\parallel = -\Omega_e - \frac{B^2}{8\pi}, \]

\[ P_\perp = -\Omega_e - M_e B + \frac{B^2}{8\pi}, \]

\[ N_e = N_p \]

\[ A / Z = 2 \]

WD composed by carbon
Z atomic number
A baryon number.

\[ N = -\frac{\partial \Omega_e}{\partial \mu} \]

\[ M = -\frac{\partial \Omega_e}{\partial B} \]
Structure equilibrium equations
Structure equations: TOV

Space-time static and spherically symmetric

\[ ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\Lambda(r)} dr^2 + r^2 d\Omega^2 \]

from Einstein equation \( G^{\mu\nu} = 8\pi G T^{\mu\nu} \)

and isotropic \( T^{\mu\nu} = \text{diag}(\varepsilon, P, P, P) \quad T_{\nu;\mu} = 0 \)

TOV

\[
\begin{align*}
\frac{dp}{dr} &= -\frac{G}{c^2} \frac{m(r)\varepsilon(r)}{r^2} \left[ 1 + \frac{p(r)}{\varepsilon(r)} \right] \left[ 1 + \frac{4\pi r^3 p}{m(r)c^2} \right] \left[ 1 - \frac{2Gm(r)}{r} \right]^{-1} \\
\frac{dm}{dr} &= \frac{4\pi r^2 \varepsilon(r)}{c^2}
\end{align*}
\]

with the boundary conditions \( M(0) = 0, P(R) = 0 \)

and the EOS determine \( P(O) = P_c \).
Mass Radius relations using TOV for magnetized/SSs/WDs
Mass-radius relation/MSSs

\[ \frac{M}{M_\odot} \text{ vs. } R \text{ [km]} \]

- \( P_\perp, B = 10^{18} \text{ G} \)
- \( P_\parallel, B = 10^{18} \text{ G} \)
- \( P_\perp, B = 10^{17} \text{ G} \)
- \( P_\parallel, B = 10^{17} \text{ G} \)

\( B_{\text{bag}} = 75 \text{ MeV fm}^{-3} \)
Mass-radius relation/MWDs

...but maximum mass for WDs are $1.44 \, M_\odot$
Anisotropic structure equations

...first attempt to obtain static structure stellar equilibrium equations for anisotropic B-dependent EoS


Anisotropic stellar structure equations

\[ ds^2 = -e^{2\Phi} dt^2 + e^{2\Lambda} dr^2 + e^{2\Omega} d\phi^2 + e^{2\Psi} dz^2 \]

\[ e^{2\Omega} = r^2 \]

Consider

\[ G^{\mu\nu} = 8\pi GT^{\mu\nu} \]

energy-momentum

\[ T^{\mu\nu} = \text{diag}(\epsilon, P_\perp, P_\perp, P_\parallel) \quad \text{with} \quad T^{\mu}_{\nu;\mu} = 0 \]

following the procedure of Trendafilova and S. A Fulling 2011 Eur JPhys 32 1663/1677 devoted to obtain static solutions of Einstein equation with cyllidrical symmetry for vacuum.

and with the main approximation that all variables depend only on \( r \)
Anisotropic stellar structure equations

\[
P'_\perp = -\Phi'(E + P_\perp) - \Psi'(P_\perp - P_\parallel),
\]

\[
4\pi e^{2\Lambda} (E + P_\parallel + 2P_\perp) = \Phi'' + \Phi'(\Psi' + \Phi' - \Lambda') + \frac{\Phi'}{r},
\]

\[
4\pi e^{2\Lambda} (E + P_\parallel - 2P_\perp) = -\Psi'' - \Psi'(\Psi' + \Phi' - \Lambda') - \frac{\Psi'}{r},
\]

\[
4\pi e^{2\Lambda} (P_\parallel - E) = \frac{1}{r} (\Psi' + \Phi' - \Lambda')
\]

\[P_\parallel \rightarrow f(E), P_\perp = f(P_\parallel)\]

Six quantities to determine \[P_\perp, P_\parallel, E, \Lambda, \Psi, \phi\]
Anisotropic stellar structure equations

Initial conditions:

\[ P_\perp (0) = P_{\perp 0} \]

\[ \Lambda (0) = 0 \]

\[ \phi (0) = \frac{1}{2} (P_{||0} + 2P_{\perp 0} + E_0) (r_0^2 - 2r_0) \]

\[ \Psi (0) = \frac{1}{2} (-P_{||0} + 2P_{\perp 0} - E_0) (r_0^2 - 2r_0) \]

\[ \phi' = \psi' = 0 \]
Metric coefficients/MSSs

B = 10^{17} G
- - - \Lambda
- - - \Phi
- - - - \Psi
B = 10^{18} G
- - - - - \Lambda
- - - - \Phi
- - - - - - \Psi
Pressures versus equatorial r/MSSs

![Graph showing pressures versus equatorial radius for different magnetic fields.](image)
To calculate the total mass of the stars we have to use the definition of mass given by Tolman 1939

$$M_T = \int \sqrt{-g} (T_0^0 - T_1^1 - T_2^2 - T_3^3) dV$$

$$M_T = \int r e^{\Phi + \Psi + \Lambda} \left( \epsilon - 2P_{\perp} - P_{\parallel} \right) dV$$

Due to our assumption that $\phi, \psi, \Lambda$ depend only on $r$ so

$$\frac{M_T}{R_{\parallel}} = 4\pi \int_0^{R_{\perp}} r e^{\Phi + \Psi + \Lambda} \left( \epsilon - 2P_{\perp} - P_{\parallel} \right) dr.$$ 

We obtain the total Mass per unit length, not the total Mass!!!
then, which information can be extract for our model?

Imposing $P_{\perp}(R_{\perp}) = 0$ to determine the equatorial radius of the star and with this $M/L$ we illustrate

- Magnetized Strange Stars
- Magnetized White Dwarfs
The important thing is to obtain a bound for the maximum magnetic field that support the MSS stars!!! $B=1.8 \times 10^{18} \text{ G}$ closer the value obtained by Virial theorem.
\[
\frac{M}{R_{\|}} \quad \text{Versus perpendicular radius}
\]

First glance shows
\[
\frac{M}{R_{\|}} \gtrsim 1.44 M_{\odot}
\]
but this value is not associated to maximum mass of the stars!!

We obtain a threshold for the magnetic field that support A WD around \( B=10^{13} \text{ G} \) the same obtained by Virial theorem.
Conclusions

1. We have obtained an “anisotropic stellar structure equations” based on cylindrical metric compatible with anisotropic produced by the magnetic field in the EoS.

2. The main approximation of our model is to consider that metric coefficients only depend on r. The price that we pay is that we can not compute total mass of the stars.
Conclusions

3. We have obtained a **MAXIMUM VALUE** of the magnetic field that can support

   Magnetic Strange Stars/Magnetic White Dwarfs

   \[
   \begin{align*}
   \text{MSS} & : 1.8 \times 10^{18} G \\
   \text{MDWs} & : 10^{13} G
   \end{align*}
   \]

4. Our model rule out Super Chandrasekhar MWD because their existence depend on to consider values of the magnetic field higher than and beyond to this value there are no stable configurations of MWD.
Our future work is addressed to improve the structure anisotropic equation taking into account the dependency on $z$, and angles in order to obtain the total mass of the stars. We are open to suggestions to improve the model.
Thank you, Gracias!!!