Quantum Gravity with Anisotropic Scaling and the Multicritical Universe

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Some references

initial papers on gravity with anisotropic scaling:

..., arXiv:0812.4287, arXiv:0901.3775, ...

brief review:


focus of this talk:

work with Tom Griffin and Charles Melby-Thompson in arXiv:1112.5660, 1211.4872, & to appear

work with Kevin Grosvenor, Tom Griffin and Ziqi Yan in arXiv:1308.5967
Fundamental physics in the 21st Century

Built on two paradigms of the 20th century:

Relativity – first special, unifying space and time via Lorentz symmetries, characterized by the speed of light $c$, 
... – then general, unifying gravity and spacetime geometry:

![Lorentz symmetries](image)

... describes the universe at large scales (black holes, big-bang, ...)

and then there is Quantum mechanics, characterized by the Planck constant $\hbar$, measuring the uncertainty between coordinates $q$ and momenta $p$; describes our worlds well at microscopic scales (everything except gravity).
Reasons for unification of QM and GR

Why to look for quantum gravity?

1. Conceptual unity of “fundamental” interactions.

There is also condensed matter (many-body physics in fixed spacetime), with fascinating “derived” or “emergent” collective phenomena.

2. History of unifications – as explanations of dimensionful constants of Nature – because Newton’s constant remains unexplained, one more revolution is left! (also – new twist to this puzzle: the cosmological constant $\Lambda$)

3. Human curiosity: Which paradigm is more fundamental? Relativity, or “quantum”? 

Early attempts to find quantum gravity

Classical gravity is described by an action principle,

\[ S_{EH} = \frac{1}{16\pi G_N} \int_M d^4 x \sqrt{g} (R - 2\Lambda), \]

which enjoys a local “gauge invariance” – under spacetime diffeomorphisms \( \text{Diff}(M) \).

So, let’s just apply techniques of relativistic quantum field theory, which worked so well for Yang-Mills and matter!

Problems with gravity: Non-renormalizable (= not “UV complete”), hence only an effective theory, predicting its own limits and eventual demise, around (or way before!) the characteristic scale, the Planck scale.
Naturalness Puzzles at the Forefront

Phenomena at low energies/long distances should follow from those at higher energies/shorter distances (basically, causality).

Made more precise in the 70’s ['t Hooft], as the Principle of (Technical) Naturalness.

A surprisingly many of the most fundamental puzzles of current fundamental physics can be phrased as a Naturalness problem:

Dark energy – in the presence of short-distance quantum fluctuations, why is the Universe so large and peaceful (=slowly evolving)? Why is the cosmological constant so small??

The hierarchy problem of particle physics – why is the scale of electroweak symmetry breaking $M_{EW} \sim 100$ GeV so much smaller than the high scale ($M_P$, $M_{GUT}$, . . .), especially in the presence of the Higgs?? (Thus puzzle made much more pressing by the recent LHC results.)
Gravity with anisotropic scaling
(also known as Hořava-Lifshitz gravity)

Field theory with anisotropic scaling \((x = \{x^i, i = 1, \ldots, D\})\):

\[x \rightarrow \lambda x, \quad t \rightarrow \lambda^z t.\]

\(z\): dynamical critical exponent – characteristic of RG fixed point.

Many interesting examples: \(z = 1, 2, \ldots, n, \ldots\)
fractions: \(3/2\) (KPZ surface growth in \(D = 1\)), \(\ldots, 1/n, \ldots\)
families with \(z\) varying continuously \(\ldots\)

Condensed matter, dynamical critical phenomena, quantum critical systems, \(\ldots\)

Goal: Extend to gravity, with propagating gravitons, formulated as a quantum field theory of the metric.
Gravity with anisotropic scaling
(also known as Hořava-Lifshitz gravity)

Evgenii Mikhailovich Lifshitz (1915 – 1985)
Comparison to Asymptotic Safety

In both, search to define gravity via a renormalization-group UV fixed point.

**Asymptotic safety:** look for relativistic, nontrivial RG fixed points. [Weinberg, ...]

**Gravity with anisotropic scaling:** looking also for nonrelativistic, often Gaussian fixed points. (in UV: leading to improved short-distance behavior of gravity; or in IR: emergent in cond-mat systems.)

Price paid for improved UV behavior: Anisotropy between space and time (or even spatial) and absence of Lorentz symmetry at short distances.

Flow between UV and IR: from $z > 1$ to $z = 1$. Lorentz symmetry must be emergent at low energies, with systematic energy-dependent Lorentz-violating corrections.
Why is this interesting?

(i) Phenomenology of gravity in our Universe, $3 + 1$ dimensions. How close can this resemble GR in IR? The multicritical universe scenario;

(ii) Gravity duals of field theories in AdS/CFT; in particular, candidates for duals of nonrelativistic field theories;

(iii) Useful also in conventional Einstein gravity, in spacetimes which are asymptotically anisotropic!

(iv) Analytic tool for understanding numerical results of lattice quantum gravity;

(v) Gravity on worldvolumes of branes;

(vi) Mathematical applications (theory of the Ricci flow);

(vii) Emergent Gaussian IR fixed points in lattice systems of condensed matter.
Update on the status of Lifshitz gravity
Example: Lifshitz scalar [Lifshitz, 1941]

Gaussian fixed point with $z = 2$ anisotropic scaling:

$$S = S_K - S_V = \frac{1}{2} \int dt \, d^D x \left\{ \dot{\Phi}^2 - (\Delta \Phi)^2 \right\},$$

($\Delta$ is the spatial Laplacian).

Compare with the Euclidean field theory

$$W = -\frac{1}{2} \int d^d x (\partial \phi)^2.$$

Shift in the (lower) critical dimension:

$$[\phi] = \frac{d - 2}{2}, \quad [\Phi] = \frac{D - 2}{2}.$$
Gravity at a Lifshitz point

**Spacetime structure:** Preferred foliation by leaves of constant time (avoids the “problem of time”, good for cosmology!).

**Fields:** Start with the spacetime metric in ADM decomposition: the spatial metric $g_{ij}$, the lapse function $N$, the shift vector $N^i$.

**Symmetries:** foliation-preserving diffeomorphisms, $\text{Diff}(M,F)$.

**Action:** $S = S_K - S_V$, with

$$S_K = \frac{1}{\kappa^2} \int dt \, d^Dx \sqrt{g} N \left( K_{ij} K^{ij} - \lambda K^2 \right)$$

where $K_{ij} = \frac{1}{N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i)$ the extrinsic curvature,

and

$$S_V = \frac{1}{\kappa^2 V} \int dt \, d^Dx \sqrt{g} N \mathcal{V}(R_{ijkl}, \nabla_i).$$
Simplest example: $z = 2$ gravity

The action is $S = S_K - S_V$, with

$$S_K = \frac{1}{\kappa^2} \int dt \, d^D x \sqrt{g} N \left( K_{ij} K^{ij} - \lambda K^2 \right)$$

and

$$S_V = \int dt \, d^D x \sqrt{g} N \left( \alpha R_{ij} R^{ij} + \beta R^2 + \ldots \right).$$

Shift in the critical dimension, as in the Lifshitz scalar:

$$[\kappa^2] = 2 - D.$$

The minimal theory with $N(t)$ has the usual number of transverse-traceless graviton polarizations, plus an extra scalar DoF, all with the dispersion relation $\omega^2 \sim k^4$.

Two special values of $\lambda$: 1 and $1/D$. 
Another example: \( z = 3 \) gravity

The action is again \( S = S_K - S_V \), with

\[
S_K = \frac{1}{\kappa^2} \int dt \, d^Dx \sqrt{g} N \left( K_{ij} K^{ij} - \lambda K^2 \right)
\]

and

\[
S_V = \int dt \, d^Dx \sqrt{g} N C_{ij} C^{ij}.
\]

where \( C^{ij} = \varepsilon^{ik\ell} \nabla_k (R^j_{\ell} - \frac{1}{4} R \delta^j_{\ell}) \) is the Cotton-York-ADM tensor. The shift of the critical dimension is

\[
[\kappa^2] = 3 - D.
\]

Anisotropic Weyl invariance would eliminate the scalar graviton classically.
Projectable and nonprojectable theory

$N, N_i$ are the gauge fields for the $\text{Diff}(M, F)$ symmetries generated by $\delta t = f(t), \delta x^i = \xi^i(t, x)$. Hence:

(1) we can restrict $N(t)$ to be a function of time only: projectable theory.

(2) or, we allow $N(t, x)$ to be a spacetime field. New terms, containing $\nabla_i N/N$, are then allowed in $S$ by symmetries: nonprojectable theory.

Spectrum: Tensor graviton polarizations, plus an extra scalar graviton. Three options for the scalar: Live with it, gap it, or eliminate it by an extended gauge symmetry.

Dispersion relation: Nonrelativistic, $\omega^2 \sim k^{2z}$, around this Gaussian fixed point.

Allowed range of $\lambda$: $0 \leq \lambda \leq 1/D$
RG flows

Assume $z > 1$ UV fixed point. Relevant deformations trigger RG flow to lower values of $z$. **Example:** Lifshitz scalar.

\[
S = \frac{1}{2} \int dt \, d^D x \left\{ \dot{\Phi}^2 - (\Delta \Phi)^2 - \mu^2 \partial_i \Phi \partial_i \Phi - m^4 \Phi^2 \right\},
\]

**Multicriticality.** New phases: modulated.

Similarly for gravity:

\[
S = \frac{1}{\kappa^2} \int dt \, d^D x \sqrt{g} N \left\{ K_{ij} K^{ij} - \lambda K^2 - \ldots - \mu^{2z-2} R - M^{2z} \right\}.
\]

**Flows in IR to $z = 1$ scaling.** In the IR regime, $S_V$ is dominated by the spatial part of Einstein-Hilbert!
Gravity on the lattice

Causal dynamical triangulations approach [Ambjørn,Jurkiewicz,Loll] to $3 + 1$ lattice gravity:

Naive sum over triangulations does not work (branched polymers, crumpled phases).

Modify the rules, include a preferred causal structure:

With this relevant change of the rules, a continuum limit appears to exist: The spectral dimension $d_s \approx 4$ in IR, and $d_s \approx 2$ in UV. Continuum gravity with anisotropic scaling: $d_s = 1 + D/z$. ([Benedetti,Henson,2009]: works in $2 + 1$ as well.)
Relevant deformations, RG flows, phases

The Lifshitz scalar can be deformed by relevant terms:

\[ S = \frac{1}{2} \int dt \, d^Dx \left\{ \dot{\phi}^2 - (\Delta \phi)^2 - \mu^2 \partial_i \phi \partial_i \phi + m^4 \phi^2 - \phi^4 \right\} \]

The undeformed \( z = 2 \) theory describes a tricritical point, connecting three phases – disordered, ordered, spatially modulated ("striped") [A. Michelson, 1976]:

![Graphical representation of the theory](image-url)
Phase structure in the CDT approach

Compare the phase diagram in the causal dynamical triangulations:

[Ambjørn et al, 1002.3298]

Note: $\varepsilon = 2$ is sufficient to explain three phases.
Possibility of a nontrivial $\varepsilon \approx 2$ fixed point in $3 + 1$ dimensions?
RG flows in gravity: $z = 1$ in IR

Theories with $z > 1$ represent candidates for the UV description. Under relevant deformations, the theory will flow in the IR. Relevant terms in the potential:

$$\Delta S_V = \int dt \, d^D x \sqrt{g} N \left\{ \ldots + \mu^2 (R - 2\Lambda) \right\}.$$ 

the dispersion relation changes in IR to $\omega^2 \sim k^2 + \ldots$

the IR speed of light is given by a combination of the couplings $\mu^2$ combines with $\kappa, \ldots$ to give an effective $G_N$.

Sign of $k^2$ in dispersion relation is opposite for the scalar and the tensor modes! Can we classify the phases of gravity? Can gravity be in a modulated phase?
Phases of gravity with anisotropic scaling

[w/ C. Melby-Thompson, K. Grosvenor]

In $2 + 1$ dimensions, take FRW, with $k = 1$. Phase diagram:

Governed by $\int \mathcal{H}_\perp \equiv (\dot{g})^2 + R^2 + \mu^2 R - 2\Lambda = 0$, the Friedmann equation.
Phases of gravity with anisotropic scaling

[w/ C. Melby-Thompson, K. Grosvenor]

In \(2 + 1\) dimensions, take FRW, with \(k = 1\). Phase diagram:

Governed by \(\int H_\perp \equiv (\dot{g})^2 + R^2 + \mu^2 R - 2\Lambda = 0\), the Friedmann equation.

cf.: CDT phase structure in \(2 + 1\) dimensions [arXiv:1111.6634, w/ C. Anderson, S. Carlip, J.H. Cooperman, R. Kommu and P. Zulkowski.]

Deconfinement of \(\int H_\perp\)?
Emergent gravity at a Lifshitz point

[Cenke Xu and P.H., arXiv:1003.0009]

These models with $z = 2$ or $z = 3$ gravitons can emerge as IR fixed points on the fcc lattice. Emergent gauge invariance stabilizes new algebraic bose liquid phases.

Recall the emergence of $U(1)$ “photons” in dimer models [Fradkin,Kivelson,Rokhsar,...]:

Lattice symmetries protect $z = 2$ or $z = 3$ in IR, forbid $G_N$. But: interacting Abelian gravity is possible!
Lifshitz holography

AdS:

\[ ds^2 = r^2(-dt^2 + dx^2) + dr^2/r^2 \]

Lifshitz:

\[ ds^2 = -r^{2z}dt^2 + r^2dx^2 + dr^2/r^2 \]

Focus in this talk:
The role of HL gravity for Lifshitz spacetime.

[T. Griffin, C. Melby-Thompson, PH]
Which theory does this space solve?

Not Einstein equations in the vacuum . . .

Two options:

(a) Keep theory relativistic, modify by inventing suitable matter; Lifshitz spacetime may be a solution when matter is excited.

(b) Modify gravity; Lifshitz spacetime may be a vacuum solution.

In both cases, ideas of anisotropic gravity play a central role.
Relativistic holography for Lifshitz space

Various matter sources possible, one of the simplest/most popular is a massive vector: [M. Taylor]

\[
S = \frac{1}{16\pi G_N} \int dt \, d^D x \, dr \sqrt{-G} (\mathcal{R} - 2\Lambda) + \frac{1}{8\pi G_N} \int dt \, d^D x \sqrt{-g} \mathcal{K}
\]

\[
- \frac{1}{4} \int dt \, d^D x \, dr \sqrt{-G} (F_{\mu\nu} F^{\mu\nu} + 2m^2 A_\mu A^\mu).
\]

To get Lifshitz with given \(z\) as solution, one must take

\[
\Lambda = \frac{1}{2} \left[ z^2 + (D - 1)z + D^2 \right], \quad m^2 = Dz, \quad \langle A_t \rangle = \frac{2(z - 1)}{z}.
\]

Also, various string/M-theory embeddings now exist. [Balasubramanian,Narayan; Gauntlett et al; . . .]
Anisotropic Weyl symmetries

Using a spacetime-dependent scaling factor $\Omega(t, x) = e^{\omega(t, x)}$, define

$$g_{ij} \rightarrow \Omega^2 g_{ij}, \quad N_i \rightarrow \Omega^2 N_i, \quad N \rightarrow \Omega^z N.$$ 

Such anisotropic Weyl transformations are compatible with foliation-preserving diffeos:

$$\text{Weyl}_z(M, \mathcal{F}) \ltimes \text{Diff}(M, \mathcal{F}).$$

Indeed, $[\delta_\omega, \delta_f, \xi^i] = \delta_f \omega + \xi^i \partial_i \omega$.

This provides the appropriate generalization of local “conformal transformations” to foliated spacetimes with anisotropic scaling.

Often, it is sufficient to have the preferred foliation and the anisotropic Weyl transformations defined only asymptotically, “near infinity.”
Anisotropic Weyl anomalies

Consider an “anisotropic CFT” in $D + 1$ dimensions, with dynamical exponent $z$.

Place it in a gravitational background. The theory may exhibit anisotropic Weyl anomaly:

$$\delta_\omega W_{\text{eff}} = \int dt \, d^D x \sqrt{g} N \omega(t, x) A(t, x).$$

What terms can appear in $A$? Question in BRST cohomology of $\text{Weyl}_z(M, \mathcal{F})$.

**Answer for $D = 2$, $z = 2$:** Assuming time reversal invariance, two independent anomaly terms (hence two “central charges”)

$$A = c_K \left( K_{ij} K^{ij} - \frac{1}{2} K^2 \right) + c_V \left[ R - \left( \frac{\nabla N}{N} \right)^2 + \frac{\Delta N}{N} \right]^2.$$
Conformal gravity at a Lifshitz point

For some fixed $z$, extend the gauge symmetry from $\text{Diff}(M, \mathcal{F})$ to $\text{Weyl}_z(M, \mathcal{F}) \ltimes \text{Diff}(M, \mathcal{F})$.

Then:

- The kinetic term $S_K$ will be invariant if $z = D$ and $\lambda = 1/D$.
- The theory is automatically nonprojectable.
- The classical theory may be invariant, but the quantum theory generally expected to develop a Weyl anomaly.
- Choice: Theory may or may not satisfy detailed balance.

Example: Conformal $z = 2$ gravity in $D = 2$ with detailed balance,

$$S = S_K = \frac{1}{\kappa^2} \int dt \, d^2x \sqrt{g} N \left( K_{ij} K^{ij} - \frac{1}{2} K^2 \right).$$
Anisotropic conformal infinity

Even if we embed Lifshitz space into relativistic gravity with matter, we need anisotropic scaling to define properly the asymptotic structure near “infinity”.

Conformal infinity [Penrose]: Rescale the bulk metric on $\mathcal{M}$

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} \equiv \Omega(t, x, r) g_{\mu\nu},$$

so that $\tilde{g}_{\mu\nu}$ now extends “past infinity” of $\mathcal{M}$. Then take $\overline{\mathcal{M}}$. For Lifshitz spacetimes with $z > 1$: Conformal infinity is a line! (This contradicts holographic expectations).

Anisotropic conformal infinity [PH & C. Melby-Thompson, 2009]: For asymptotically foliated spacetimes, one can use anisotropic Weyl transformations. Anisotropic conformal infinity of Lifshitz spacetime is of codimension one, with a preferred nonrelativistic foliation. (As expected from holography.)
Holographic renormalization

Holographic correspondence:

\[ \mathcal{Z}_{\text{bulk}}[\Phi|_{\partial \mathcal{M}} = \Phi_0] = \left\langle \exp \left( \int_{\partial \mathcal{M}} \Phi_0 \mathcal{O} \right) \right\rangle_{\text{CFT}}. \]

In the classical gravity limit, this implies

\[ W_{\text{eff}}[\Phi_0] = -S_{\text{on-shell}}[\Phi_0]_{\text{gravity}}. \]

The action diverges, requires (holographic) regularization and renormalization:
Holographic renormalization

Holographic correspondence:

\[ Z_{\text{bulk}} [\Phi|_{\partial M} = \Phi_0] = \langle \exp \left( \int_{\partial M} \Phi_0 \mathcal{O} \right) \rangle_{\text{CFT}}. \]

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The action diverges, requires (holographic) regularization and renormalization:
Counterterms: Hamilton-Jacobi theory

[de Boer, Verlinde, Verlinde; Skenderis et al; Ross & Saremi; . . . ]

On-shell action \( S[g_{ij}, N_{i}, N; . . . ] \), as a functional of boundary fields, satisfies HJ equation in radial evolution.

In gravity, this means
\[
\mathcal{H}_{r} \left( \pi_{\alpha\beta} = \frac{\delta S}{\delta g_{\alpha\beta}} \right) = 0
\]
(plus supermomentum constraints).

Parametrize
\[
S[g_{ij}, N_{i}, N] = \frac{1}{16\pi G_{N}} \int dt d^{D}x \sqrt{g_{N}} \mathcal{L}.
\]

Note: \( \mathcal{L} \) is divergent, expand asymptotically as
\[
\mathcal{L} = \ldots + \frac{\mathcal{L}^{(4)}}{\varepsilon^{4}} + \frac{\mathcal{L}^{(2)}}{\varepsilon^{2}} + \tilde{\mathcal{L}}^{(0)} \log \varepsilon + \mathcal{L}^{(0)} + \mathcal{O}(\varepsilon^{2}).
\]

Plug into HJ equation, which gives a recursive relation among counterterms.
Counterterms for \( z = 2 \) in \( D = 2 \):
Conformal Lifshitz gravity

Holographic recursion relations for counterterms can be solved for general \( D \) and \( z \), at least in principle.

In the first nontrivial conformal case, \( D = 2 \) and \( z = 2 \), they give:

\[
\mathcal{L}^{(4)} = 6, \quad \mathcal{L}^{(2)} = \frac{1}{2} R + \frac{1}{4} \left( \frac{\nabla N}{N} \right)^2, \quad \tilde{\mathcal{L}}^{(0)} = K_{ij} K^{ij} - \frac{1}{2} K^2.
\]

The recursion relations imply \( \delta_D \mathcal{L}^{(0)} \sim \tilde{\mathcal{L}}^{(0)} \).

Since \( \mathcal{L}^{(0)} \) is the renormalized action, and \( \delta_D \) (=the radial evolution operator) is the anisotropic Weyl rescaling, \( \tilde{\mathcal{L}}^{(0)} \) gives the anisotropic Weyl anomaly.

This anomaly takes the form of \( z = 2 \) conformal Lifshitz gravity action in detailed balance, at the anisotropic boundary.
Holographic anomaly in detailed balance

Note that of the two possible central charges, only $c_K$ is generated in this holographic theory. What distinguishes $c_K$ from $c_V$? The $c_K$ anomaly satisfies the detailed balance condition.

This was further verified for bulk gravity coupled to scalars $X^I$. In addition to the kinetic piece,

$$\frac{1}{\kappa^2} \int dt \, d^2x \sqrt{g} N \left( K_{ij} K^{ij} - \frac{1}{2} K^2 \right) + \frac{1}{2} \int dt \, d^2x \frac{\sqrt{g}}{N} \left( \dot{X}^I - N^i \partial_i X^I \right)^2,$$

the anomaly now develops also a potential part,

$$\int dt \, d^2x \sqrt{g} N \left\{ (\Delta X^I)^2 + \frac{\kappa^2}{2} T_{ij}(X) T^{ij}(X) \right\}.$$

This is in detailed balance, with $W = \frac{1}{2} \int d^2x \sqrt{g} \partial_i X^I \partial^i X^I$. 
Field-theory examples with $z = 2$, $D = 2$

Lifshitz scalar field in gravitational background:

$$S = \frac{1}{2} \int dt \, d^2 x \sqrt{g} \left\{ \frac{1}{N} \left( \dot{\Phi} - N^i \nabla_i \Phi \right)^2 - N (\Delta \Phi)^2 \right\}.$$

Classically anisotropically Weyl invariant with $z = 2$, $\delta \Phi = 0$.

Quantum Weyl anomaly: [de Boer et al., also us]

Calculated using $\zeta$-function regularization,

$$c_K = \frac{1}{32\pi}, \quad c_V = 0.$$

Only one non-zero central charge; anomalies in detailed balance!
Detailed balance from holographic recursion

Why in the world should the holographic Weyl anomaly satisfy detailed balance?

**Answer:** This follows from the holographic recursion relation among the counterterms!

Indeed, the HJ equation – expanded order by order in the conformal dimension – implies that

\[ \tilde{\mathcal{L}}^{(0)} \sim \left( \frac{\delta \mathcal{L}^{(2)}}{\delta g_{\alpha\beta}} \right)^2. \]

This is precisely the condition of detailed balance, with the quadratic counterterm \( \mathcal{L}^{(2)} \) playing the role of \( W \)!
More QFT examples with $\varepsilon = 2$, $D = 2$

We saw that the minimal Lifshitz scalar has one central charge $c_K \neq 0$ but the other one $c_V = 0$.

Are there theories with both independent central charges?
(if not, then we don’t need to look for their holographic description . . .)
More QFT examples with $z = 2$, $D = 2$

We saw that the minimal Lifshitz scalar has one central charge $c_K \neq 0$ but the other one $c_V = 0$.

Are there theories with both independent central charges? (if not, then we don’t need to look for their holographic description . . . ) Yes!

$$S = \frac{1}{2} \int dt \, d^2x \sqrt{g} \left\{ \frac{1}{N} \left( \dot{\Phi} - N_i \nabla_i \Phi \right)^2 - N (\Delta \Phi)^2 \right\}$$

$$- \frac{e^2}{2} \int dt \, d^2x \sqrt{g} N \left[ R - \left( \nabla \Phi \right)^2 + \frac{\Delta \Phi}{N} \right]^2 \Phi^2.$$  

This non-minimally coupled theory has $c_K = 1/(32\pi)$ and $c_V = -e^2/(8\pi)$.

Now that we know that theories with independent $c_K$, $c_V$ exist, what is the holographic dual of $c_V$?
Holography with the general anomaly?

- The simplest relativistic system does not work.
- If looking for a more complicated relativistic model:
  Need to relax the holographic recursion between counterterms,

\[ \tilde{\mathcal{L}}^{(0)} \neq \left( \frac{\delta \mathcal{L}^{(2)}}{\delta g_{\alpha \beta}} \right)^2. \]

- But: it turns out that Lifshitz gravity works, in the vacuum!

Disclaimer: That does not imply that one cannot do this with relativistic gravity coupled to matter; indeed, the boundary between relativistic and nonrelativistic is somewhat fuzzy.)
Lifshitz gravity for Lifshitz holography

Requires nonprojectable theory (because we anticipate anisotropic Weyl transformations on the codimension-one boundary, hence $N$ near boundary must depend on spacetime).

Lifshitz spacetime now foliated not just asymptotically, but everywhere (in the bulk). A single, codimension-one foliation by leaves of constant $t$ (multiple & nested foliations also possible, not studied here).

At low energies, just as in GR:

- Anisotropic gravity will be dominated by $z = 1$ terms.
- We neglect the small corrections due to all other higher-derivative terms, including the $z > 1$ terms in $S_V$. 
Low-energy effective theory

The low-energy action will be similar to Einstein-Hilbert, with several important differences:

\[ S = \frac{1}{\kappa^2} \int dt \, d^D x \, dr \sqrt{G} N \left( K_{ab} K^{ab} - \lambda K^2 \right) \]

\[ + \frac{1}{\kappa^2} \int dt \, d^D x \, dr \sqrt{G} N \left[ R - 2\Lambda - \frac{\alpha^2}{2} \left( \nabla_a N \right)^2 \right]. \]

We have two additional low-energy couplings compared to GR: \( \lambda \) and \( \alpha \)

(one will be fixed by choosing \( z \) for the vacuum, the other is free to give a missing central charge in the anisotropic Weyl anomaly).
Lifshitz as the vacuum solution

Require now that the Lifshitz spacetime with chosen value of $z$ is a solution of the vacuum EoM.

This determines the cosmological constant (essentially, by our choice of scale),

$$\Lambda = -\frac{D(D + 1)}{2} - \frac{(z - 1)(z + 2D)}{2},$$

and fixes one of the two new couplings,

$$\alpha^2 = \frac{2(z - 1)}{z},$$

leaving $\lambda$ undetermined.

Main point: Lifshitz holography for nonrelativistic QFTs is naturally done with Lifshitz gravity in the bulk.
Multicritical Symmetry Breaking

[1308.5967, w/ Tom Griffin, Kevin Grosvenor, Ziqi Yan]

Global internal symmetry breaking leads to Nambu-Goldstone modes. Phenomenon is remarkably universal, across many fields dealing with many-body systems.

But how many NG modes, and what is their low-energy dispersion relation?

Relativistic case: All questions answered by Goldstone’s theorem. (One NG per broken generator, gapless=massless, $z = 1$ dispersion $\omega = k_z$.)

Nonrelativistic case: Classify NG modes by classifying their low-energy effective field theories [Murayama & Watanabe, 2013].

Let’s focus, for definiteness, on systems with Lifshitz symmetries. Write down possible EFT’s for NG modes $\pi^I$. 
Classification of NG Modes

[Murayama&Watanabe]: The EFTs are

\[ S = \int dt \, dx \left( \omega I(\pi) \dot{\pi}^I + g_{IJ} \dot{\pi}^I \dot{\pi}^J - h_{IJ} \partial_i \pi^I \partial_i \pi^J + \ldots \right). \]

Hence, this yields two types of NG modes:

- **Type A**, \( z = 1 \) dispersion \( \omega = ck \) (if unpaired by \( \omega \), no T-reversal breaking); as in the relativistic case, one Type A NG mode per one broken generator.

- **Type B**, dispersion \( \omega \sim k^2 \), associated with a pair of broken symmetry generators (as paired by \( \omega \)). No T-reversal symmetry. Otherwise, fine tuning.
Naturalness of Slow NG Modes

[1308.5967, w/ Tom Griffin, Kevin Grosvenor, Ziqi Yan]:
Higher \( z \) are technically Natural!

\[
S = \int dt \, dx \left( \omega_I(\pi) \dot{\pi}^I + g_{IJ} \dot{\pi}^I \dot{\pi}^J - c^2 h_{IJ} \partial_i \pi^I \partial_i \pi^J 
- G_{IJ} \partial^2 \pi^I \partial^2 \pi^J + \ldots \right).
\]

Turns out \( c^2 \) can be Naturally small, or even zero.

A new symmetry at play: the "polynomial shift",

\[
\pi^I(t, x^i) \rightarrow \pi^I(t, x^i) = a_i^I x^i x^j + a_j^I x^j + a_0.
\]

Refined classification of Natural NG modes:

- Type A tower of multicritical NG modes with \( z = 1, 2, \ldots \),
- Type B tower of multicritical NG modes with \( z = 2, 4, \ldots \),

until one hits against the multicritical Coleman Mermin-Wagner theorem, at \( z = D \). Also, hierarchies of \( z \)'s are possible!
Now, the Multicritical Universe

A hep-ph oriented scenario for “minimal” and “traditional” (= QFT, RG) reconciliation of particle physics and gravity, in a framework that can potentially be UV complete.

Ingredients

1. Particle side: the standard model, or your favorite BSM extension. Lorentz invariant. Already UV complete.

2. Gravity side: multicritical gravity, with anisotropic scaling in UV (= “HL gravity”). Flows to isotropic scaling in IR.

Robust consequences:

Anisotropic scaling communicated to the particle sector only via universal coupling to gravity.

Generic, high-energy Lorentz violations, also induced (and suppressed) by this irrelevant coupling.
September 2011, at CERN ...

\[ \frac{\nu - c}{c} = (2.48 \pm 0.28 \pm 0.30) \times 10^{-5} \text{ at } E_\nu = 17.5\text{GeV} \]
$v - c \over c = (2.48 \pm 0.28 \pm 0.30) \times 10^{-5}$ at $E_\nu = 17.5$GeV

September 2011, at CERN ...
Now ...

Everybody outside, celebrating the Higgs boson discovery!
Examples of model building in the Multicritical Universe

a. Sequestered Lifshitz gravity

Take SM, couple weakly to Lifshitz gravity (the coupling suppressed by $G_N$).

In the absence of the coupling, Lorentz symmetry in the matter sector is exact, but the coupling yields systematic high-energy effects of Lorentz violations.

Classical scales of gravity in UV: $\sim M$. Overall coupling: $\kappa$.

UV-IR relation: $x^0 = ct$, with $c = M^2$.

Newton’s constant: $G_N \sim \frac{1}{M_P^2} \sim \left(\frac{\kappa}{M}\right)^2$. 
b. Multicritical supersymmetry

Nonrelativistic, short-distance susy algebra at $\mu = 0$:

$$\{Q, \bar{Q}\} = H + \mu \gamma^i P_i$$

c. Extra dimensions, à la RS

Take, for example, $z = 4$ gravity in $4 + 1$ dimensions, with $\mathbb{R} \times H_4$ as solution.

Place the (B)SM at a flat relativistic brane.

d. A hybrid SUSY/non-SUSY scenario

Take MSSM, or your favorite (relativistic) supersymmetric extension of SM. Couple weakly to Lifshitz gravity.

Susy breaking (and small violations of Lorentz invariance) induced by this coupling. (A new twist on the gravity-mediated susy breaking.)
Conclusions

The map of the new continent of gravity with anisotropic scaling is getting more precise, with diverse applications.

- intimately connected to active areas of condensed matter and non-equilibrium systems.
- natural for holographic duals of non-relativistic field theories.
- the multicritical universe: a new paradigm for coupling gravity to matter, of the standard model and beyond;
- a systematic framework for addressing issues of Lorentz violations at high energies in our Universe,
- unification or “multification”? alternative to the multiverse?