Measurement of $e^+e^- \rightarrow$ hadrons cross sections at BABAR, and implications for the muon $g-2$

Ray F. Cowan
Representing the BABAR Collaboration
Outline

• Introduction
• The $(g-2)_{\mu}$ anomalous magnetic moment
• Hadronic vacuum polarization (HVP) corrections
• Initial-state radiation (ISR)
• BaBar’s recently updated measurements
• Results and implications
• Future Prospects
• Summary
Introduction

• The magnetic moment of a current distribution confined to a plane loop, but otherwise arbitrary, is

\[ \vec{m} = \frac{I}{2c} \int \vec{x} \times d\vec{r} \]

• For charges \( q_i \) with velocities \( \vec{v}_i \) and angular momentum \( L_i \), then

\[ \vec{m} = \frac{1}{2c} \sum q_i (\vec{x}_i \times \vec{v}_i) = \sum \frac{q_i}{2M_i c} \vec{L}_i \]

• If all the particles have the same ratio \( q_i/M_i \), then

\[ \vec{m} = \frac{e}{2Mc} \sum \vec{L}_i = \frac{e}{2Mc} \vec{L} \]

  – This is the classical connection between angular momentum and magnetic moment
  – But it fails when applied to intrinsic moments of leptons with spin \( s \)

\[ \mu_l = g_l \frac{e}{2M_l c} s, \quad a_l \equiv \frac{g_l - 2}{2} = \frac{\mu_l}{\mu_B} - 1 \]

  – \( g_l = 1 \) for orbital angular momentum
  – But is \( 2(1 + a_l) \) for \( e \) and \( \mu \)
History

• In 1926 Goudsmit and Uhlenbeck introduced the idea of electron spin to help explain the anomalous Zeeman effect (anomalous splitting of spectral lines in an external magnetic field)
  – This was pre-Dirac
  – It explained the anomalous Zeeman splitting
  – But left the fine structure splitting off by a factor of 2

• In 1927 Thomas showed that the extra factor of 2 arose from relativistic kinematic effects
  – Called the Thomas precession

• A full relativistic treatment by Dirac gave the value $g = 2$ exactly
Radiative corrections

• If \( g \) were exactly 2
  – Leptons in a magnetic field would show no longitudinal polarization change

• Radiative corrections change this
  – Provide sensitivity to loop corrections and therefore to presence of unknown particles

• First observation of effect of \( a_e \) is by Nafe, Nelson, and Rabi in 1947
  – In a difference between measured and computed values of both \( \mu_H \) and \( \mu_D \) of \((2.6 \pm 0.5) \times 10^{-3}\)
  – Yields \( a_e = 0.00118 \pm 0.00003 \)

• Prediction by Schwinger in 1948 (to first order)
  – \( a_e^{\text{theory}} = \frac{\alpha}{2\pi} = 0.001162 \)

**Experiment**

Nafe et al., *Phys.Rev.* 71, 914 (1947)

There is clearly an important difference between the measured and calculated values of \( \nu_H \) and \( \nu_D \) of about 0.26 percent compared with the probable error of the calculated value of 0.05 percent. The difference is five times greater than the claimed probable error in the natural constants. Whether the failure of theory and experiment to agree is because of some unknown factor in the theory of the hydrogen atom or simply an error in the estimate of

**Theory**

Schwinger, *Phys.Rev.* 73, 416 (1948)

The simplest example of a radiative correction is that for the energy of an electron in an external magnetic field. The detailed application of the theory shows that the radiative correction to the magnetic interaction energy corresponds to an additional magnetic moment associated with the electron spin, of magnitude \( \delta \mu / \mu = \left( \frac{3}{2} \pi \right) e^2 / hc = 0.001162 \). It is indeed gratifying that recently acquired experimental data confirm this prediction. Measurements
How to measure $a_i$

• Since the anomalous moment $a_i = (g-2)/2$ is very small
  – Longitudinal polarization of a beam in a magnetic field changes relatively slowly
  – Use this rate of change to measure $a_i$ where $l = e$ or $\mu$

• Provides one of most stringent tests of QED
  – Along with calculation of the Lamb shift
  – Confirming evidence for QED and gauge-theory based Standard Model

• Electrons
  – Use $e^-$ from $\beta$ decay, look at changes in asymmetry of Mott scattering
  – Use positronium ($e^+$) annihilation photons, look at angular distributions in a magnetic field
  – $a_e = (1159652180.76 \pm 0.27) \times 10^{-12}$
    • About 0.1 parts per billion accuracy
    • [RPP, Phys. Rev. D86, 010001 (2012)]

• Muons
  – Use muons stored in a magnetic field
  – Use angular distribution of decay electron w.r.t. direction of the circulating muon
  – $a_\mu$ has been measured repeatedly over many years
  – Including at Nevis (1960), CERN (1960s–1970s), and Brookhaven (E821, 2001)
  – Current value: $a_\mu = (11659208.0 \pm 5.4\text{(stat)} \pm 3.3\text{(syst)}) \times 10^{-10}$ (0.54 ppm)
    • [Bennett et al., Phys.Rev. D 73, 072003 (2006)]
Brookhaven experiment E821

- Protons from the AGS hit target
  - Producing pions
  - Injecting muons at 3.09 GeV/c ($\gamma_{\text{magic}} = 29.3$)
    - Avoids effect of electrostatic focusing fields
      \[
      \omega_a = -\frac{e}{m_\mu} \left[ a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1}) \frac{\vec{B} \times \vec{E}}{c} \right]
      \]
    - Magnetic field 1.45 T
  - Polarization precesses at different frequency than spin
    - $\omega_a = (\omega_s - \omega_c) = a_\mu eB/m_\mu$ at “magic” momentum
  - Muons decay to electrons
    - Strongly correlated to muon spin direction
    - Electrons above an energy threshold are detected in calorimeters inside the ring
  - Electron direction and energy in lab related to muon polarization
    - Number of $e^-$ in lab modulates at frequency $\omega_a$

$= 0$ at magic momentum
How to calculate $a_{\mu}$

- Standard Model calculation
  
  $a_{\mu}(\text{SM}) = a_{\mu}(\text{QED}) + a_{\mu}(\text{weak}) + a_{\mu}(\text{had})$
  
  \begin{align*}
  - a_{\mu}(\text{QED}) &= (116584718.10 \pm 0.15) \times 10^{-11} \\
  - a_{\mu}(\text{weak}) &= (154 \pm 2) \times 10^{-11} \\
  - a_{\mu}(\text{had}) &= (6930 \pm 49) \times 10^{-11}
  \end{align*}

- Comparison with experiment shows 3.6\sigma discrepancy
  
  \begin{align*}
  - a_{\mu}(\text{SM}) &= 116591802 \pm 49 \times 10^{-11} \\
  - a_{\mu}(\text{exp}) &= 116592089 \pm 63 \times 10^{-11}
  \end{align*}

- This is a promising place to look for new particles that could contribute to the SM calculation

**Engel et al., PRD 86, 037502 (2012); Davier et al., EPJC 71, 1515 (2011)**

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These contributions are very small for $a_e$: reduced by a factor of $(m_e/m_\mu)^2$
Higher order terms

• One graph of each order given as example out of many:
• Full QED 5th-order (in $\alpha$) calculation
Theory calculation of $a_\mu$

- **As of May 2009**

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED</td>
<td>116 584 71.81</td>
<td>± 0.02</td>
</tr>
<tr>
<td>Leading hadronic vacuum polarization (VP)</td>
<td>690.30</td>
<td>± 5.26</td>
</tr>
<tr>
<td>Sub-leading hadronic vacuum polarization</td>
<td>-10.03</td>
<td>± 0.11</td>
</tr>
<tr>
<td>Hadronic light-by-light</td>
<td>11.60</td>
<td>± 3.90</td>
</tr>
<tr>
<td>Weak (incl. 2-loops)</td>
<td>15.32</td>
<td>± 0.18</td>
</tr>
<tr>
<td><strong>Theory</strong></td>
<td><strong>11659179.00</strong></td>
<td>± 6.46</td>
</tr>
<tr>
<td><strong>Experiment</strong></td>
<td><strong>11659208.00</strong></td>
<td>± 6.30</td>
</tr>
<tr>
<td>Exp − theory</td>
<td>+29.00</td>
<td>± 9.03</td>
</tr>
</tbody>
</table>

**Assuming Gaussian statistics, a 3.2 $\sigma$ discrepancy.**

*Jegerlehner et al., Phys.Rept. 447, 1110 (2009)*
Hadronic vacuum polarization (HVP)

- Quark loops are not computable from first principles in QCD
- Vacuum polarization effects create an energy-dependent running charge
  \[ e^2 \rightarrow e^2 / \left[ 1 + \left( \Pi'(k^2) - \Pi'(0) \right) \right] \]
- Dispersion relation from analyticity
  \[ \Pi'(k^2) - \Pi'(0) = \frac{k^2}{\pi} \int_0^\infty \frac{\text{Im} \Pi'(s)}{s(s - k^2 - i\epsilon)} ds \]
- Optical theorem (unitarity)
  \[ \text{Im} \Pi'(s) = \alpha(s) R_{\text{had}}(s)/3 \]
- Where
  \[ R_{\text{had}}(s) = \sigma_{\text{had}} \frac{3s}{4\pi\alpha(s)} = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \]
Hadronic vacuum polarization (HVP)

- Putting the pieces together, we find the “dispersion integral”

\[
a_\mu^{\text{had}} = \left( \frac{\alpha m_\mu}{3\pi} \right)^2 \int \frac{R_{\text{had}}(s) \hat{K}(s)}{s^2} ds
\]

- Use pQCD to evaluate this integral above \( E_{\text{cut}} = 1.8 \text{ GeV} \)
- Use hadronic cross-section data from threshold up to \( E_{\text{cut}} \)
- So the largest uncertainty in \( a_\mu \) (theory) comes from experimentally measured \( R_{\text{had}}(s) \)
ISR cross-section measurements

- Makes optimal use of the available luminosity
- Covers the energy range with the same detector conditions and the same analysis
- At an asymmetric $e^+e^-$ collider
  - Efficiency remains good down to threshold
- Select events with high energy ISR photon
  - At large angle
- ISR photon is opposite hadrons in C.M.
  - High acceptance for boosted hadrons from threshold
- ISR luminosity determined by $e^+e^- \rightarrow \gamma_{ISR}\mu^+\mu^-$
- When the full final state is observed ($\gamma +$ hadrons)
  - The over-constrained kinematic fit provides strong background and noise rejection
- The radiator function $W(s,x)$ is
  - The density of probability to radiate a photon of energy $E_\gamma = x\sqrt{s}/2$
  - A known function

\[
\frac{d\sigma(e^+e^- \rightarrow f\gamma)}{ds'}(s') = \frac{2m}{s} W(s, x) \sigma(e^+e^- \rightarrow f)(s')
\]

\[x = 2E_\gamma/\sqrt{s} = 1 - \frac{s'}{s}\]

Established ISR program at **BABAR**

- Broad ISR program for a precise low-energy measurement of
  \[ R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \]

- Now adding \( K^+K^- \), \( K_SK_L \), \( K_SK_{S/L}\pi^+\pi^- \), \( K_SK^+K^- \) channels and updating \( p\bar{p} \)

- Measure \( \sigma(e^+e^- \rightarrow X) \) versus \( m_{\gamma^*} = m_X = E_{\text{CM}} = \sqrt{s'} \)

- Babar covers the complete set of significant exclusive channels

- Sum of exclusive channels more precise than an “inclusive” \( \gamma_{ISR} \)+hadrons measurement due to worse energy resolution for photons
**BABAR Datasets**

- PEP-II at SLAC is not only a $B$ factory but also a charm and $\tau$ factory
  - $470 \times 10^6$ $B\bar{B}$ pairs
  - $690 \times 10^6$ $c\bar{c}$ pairs
  - $500 \times 10^6$ $\tau^+\tau^-$ pairs

- **Initial-state radiation (ISR) events:** access to low-energy $e^+e^-$ hadronic cross sections

<table>
<thead>
<tr>
<th>Sample</th>
<th>Integrated $\mathcal{L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Upsilon(4S)$</td>
<td>$433$ fb$^{-1}$</td>
</tr>
<tr>
<td>$\Upsilon(3S)$</td>
<td>$30.2$ fb$^{-1}$</td>
</tr>
<tr>
<td>$\Upsilon(2S)$</td>
<td>$14.5$ fb$^{-1}$</td>
</tr>
<tr>
<td>Off-peak</td>
<td>$54$ fb$^{-1}$</td>
</tr>
<tr>
<td>Scan</td>
<td>$3.9$ fb$^{-1}$</td>
</tr>
</tbody>
</table>
Previous BaBar ISR results

\[ K^+K^-\eta, K^+K^-\pi^0, K^0K^\pm \pi^\mp \]
\[ 2(\pi^+\pi^-)\pi^0, 2(\pi^+\pi^-)\eta, K^+K^-\pi^+\pi^-\pi^0, K^+K^-\pi^+\pi^-\eta \]
\[ K^+K^-\pi^+\pi^-, K^+K^-\pi^0\pi^0, K^+K^-K^+K^- \]
\[ \Lambda\Lambda, \Lambda\Sigma^0, \Sigma^0\Sigma^0 \]
\[ 3(\pi^+\pi^-), 2(\pi^+\pi^-\pi^0), K^+K^-2(\pi^+\pi^-) \]

\[ \overline{pp} \]
\[ 2(\pi^+\pi^-), K^+K^-\pi^+\pi^-, K^+K^-K^+K^- \]
\[ \pi^+\pi^-\pi^0 \]

- First measurements
- ISR \( \gamma \) tagging \( \Rightarrow \) efficient background rejection
- Unprecedented accuracy:

\[ a_\mu(\leq 1.8 \text{ GeV}) \]

\[ 2(\pi^+\pi^-) \]
\[ 3(\pi^+\pi^-) \]
\[ 2(\pi^+\pi^-\pi^0) \]

\[ 232 \text{ fb}^{-1}, 89 \text{ fb}^{-1} \at 10.6 \text{ GeV} \]

Channel: $e^+e^- \rightarrow \pi^+\pi^-(\gamma)\gamma$

- Systematics controlled at 0.1% level
- ISR photon measured in the EMC
  - Therefore at large angle
- Require good quality tracks and particle id (PID)
- Kinematic fit uses ISR photon direction
  - Includes up to one additional photon
- All efficiencies taken from data
  - Trigger, filter, tracking, PID, fitting
- Use of measured ratio of $\pi\pi$ to $\mu\mu$ cross sections
  - Cancellation of $e^+e^-$ luminosity, additional ISR, VP, ISR efficiency
- Correct for FSR in $\mu\mu$ and for ISR plus additional FSR
  - Both calculated in QED, checked in data

\[
R_{\exp}(s') = \frac{\sigma[\pi\pi\gamma(\gamma)](s')}{\sigma[\mu\mu\gamma(\gamma)](s')} = \frac{\sigma^0[\pi\pi(\gamma)](s')}{(1 + \delta_{\text{FSR}}^{\mu\mu})\sigma^0[\mu\mu(\gamma)](s')} = \frac{R(s')}{(1 + \delta_{\text{FSR}}^{\mu\mu})(1 + \delta_{\text{add,FSR}}^{\mu\mu})}
\]
Cross section for $e^+e^- \to \pi^+\pi^-(\gamma)\gamma$ [2$m_{\pi}$, 1.8 GeV]

Bare cross section for $\sigma(e^+e^- \to \pi^+\pi^-)$, VP removed, 232 fb$^{-1}$

- $a_\mu(\pi\pi) = (514.1 \pm 2.2 \pm 3.1) \times 10^{-10}$
  - Very good precision down to threshold

- Precision similar to prior $e^+e^-$ average
  - $a_\mu(\pi\pi) = (503.5 \pm 4.5) \times 10^{-10}$

- The change is
  - $\Delta_{\pi\pi} = + (10.6 \pm 5.9) \times 10^{-10}$
  - 1.7$\sigma$ higher value

Cross section for $e^+e^- \rightarrow K^+K^-(\gamma)$

- Analysis similar to $\pi^+\pi^-$
  - Same NLO ISR, data sample, event generators
  - Data driven, step-by-step evaluation of $\varepsilon_{\text{data}}/\varepsilon_{\text{MC}}$
  - Resolution unfolding
    - Very little FSR for kaons
    - Malescu, arXiv:0907.3791

Bare cross section for $\sigma(e^+e^- \rightarrow K^+K^-(\gamma))$
Including FSR, $J/\psi$ and $\psi(2S)$ removed

Cross section for $e^+e^- \rightarrow K^+K^- (\gamma)$

- Unprecedented precision in the $\varphi$ region of $7.3 \times 10^{-3}$
  - From 1.01 to 1.03 GeV
- Dispersion integral for $a_\mu (KK, \text{LO})$ below 1.8 GeV
  - $a_\mu (KK, \text{LO}) = (22.93 \pm 0.18_{\text{stat}} \pm 0.22_{\text{syst}} \pm 0.13_{\text{VP}}) \times 10^{-10}$
  - Improved precision over previous average by 2.7
- Previous combination
  - $(21.63 \pm 0.27_{\text{stat}} \pm 0.68_{\text{syst}}) \times 10^{-10}$
- Difference
  - $\Delta_{KK} = +(1.30 \pm 0.79) \times 10^{-10}$


Recent lower-order measurements by *BABAR*

- LO: hadronic final state
- ISR luminosity computed from MC

<table>
<thead>
<tr>
<th></th>
<th>$\mathcal{L}$ (fb$^{-1}$)</th>
<th>$\sqrt{s}_{\text{min}}$ (GeV)</th>
<th>$\sqrt{s}_{\text{max}}$ (GeV)</th>
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<tbody>
<tr>
<td>$\pi^+\pi^--\pi^+\pi^-$</td>
<td>454</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>$K^+K^--\pi^+\pi^-$</td>
<td>454</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>$K^+K^-\pi^0\pi^0$</td>
<td></td>
<td>4.0</td>
<td></td>
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<tr>
<td>$K^+K^-K^+K^-$</td>
<td></td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>$\bar{p}p$</td>
<td>454</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>$\bar{p}p$</td>
<td>469</td>
<td>3.0</td>
<td>6.5</td>
</tr>
<tr>
<td>$K_S^0K_L^0$</td>
<td>469</td>
<td>2.3</td>
<td>preliminary</td>
</tr>
<tr>
<td>$K_S^0K_L^0\pi^+\pi^-$</td>
<td></td>
<td>4.0</td>
<td></td>
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<tr>
<td>$K_S^0K_L^0\pi^+\pi^-$</td>
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<td>4.0</td>
<td></td>
</tr>
<tr>
<td>$K_S^0K_S^0K^+K^-$</td>
<td></td>
<td>4.5</td>
<td></td>
</tr>
</tbody>
</table>

Blue: $\bar{p}p$ with photon undetected

First measurements
Cross section for $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\gamma$

- Below $E_{\text{cut}} = 1.8$ GeV
  - $a_\mu(\pi\pi\pi\pi) = (13.64 \pm 0.03_{\text{stat}} \pm 0.36_{\text{syst}}) \times 10^{-10}$
  - 2.6 times improvement over world average
- Previous average
  - $a_\mu(\pi\pi\pi\pi) = (13.35 \pm 0.10_{\text{stat}} \pm 0.43_{\text{syst}} \pm 0.29_{\text{syst,common}}) \times 10^{-10}$
- Difference
  - $\Delta_{\pi\pi\pi\pi} = +(0.29 \pm 0.63) \times 10^{-10}$
- Supersedes our previous 2005 measurement $2(\pi\pi)$


$e^+e^- \rightarrow K^+K^-\bar{h}h\gamma$

$K^+K^-\pi^+\pi^-$

$K^+K^-\pi^0\pi^0$

$K^+K^-K^+K^-$

454 fb$^{-1}$


Statistical uncertainties only
Latest results: cross sections for ISR with $K_S K_S$, $K_S K_L$, $+ K_S K_L$ ...

$e^+ e^- \rightarrow K_S^0 K_L^0$

Fit including $\phi(1680)$

$e^+ e^- \rightarrow K_S^0 K_L^0 \pi^+ \pi^-$

469 fb$^{-1}$

Statistical uncertainties only

$e^+ e^- \rightarrow K_S^0 K_S^0 \pi^+ \pi^-$

$e^+ e^- \rightarrow K_S^0 K_S^0 K^+ K^-$
Overview of all *BABAR* ISR cross sections

- As of October 2013
  - The $\pi^+\pi^-\pi^0\pi^0$ result is preliminary: arXiv:0710.3455

Courtesy Fedor V. Ignatov
SM comparison with BNL E821

- Plot includes BABAR data up to 2011
  - But not latest (2012 & 2013)
- Doesn’t include latest KLOE $\pi\pi/\mu\mu$ measurement
- Uncertainty rescaling is used where experimental combinations are inconsistent
- (6) solves problem with discrepancy in $e^+e^-$ vs. $\tau$ spectral functions
Effect of newer $BABAR$ measurements

- The most recent $BABAR$ measurements are not taken into account on previous slide
- Biggest change is from $K^+K^-$ and $2(\pi^+\pi^-)$ hadronic cross sections below 1.8 GeV
  
  \[ \Delta = \Delta a_{\mu}^{K^+K^-} + \Delta a_{\mu}^{2(\pi^+\pi^-)} = 16 \times 10^{-11} \]

  — Gross estimate using typical values ($\times 10^{-11}$)

<table>
<thead>
<tr>
<th></th>
<th>SM</th>
<th>Exp</th>
<th>SM - Exp</th>
<th>significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>without</td>
<td>$-300 \pm 60$</td>
<td>$0 \pm 63$</td>
<td>$-300 \pm 87$</td>
<td>3.4</td>
</tr>
<tr>
<td>with</td>
<td>$-284 \pm 60$</td>
<td>$0 \pm 63$</td>
<td>$-284 \pm 87$</td>
<td>3.2</td>
</tr>
</tbody>
</table>

  — This crude exercise overestimates the shift
    - Which in any case is $< 0.2\sigma$
Current summary of \textit{BABAR} measurements

A vigorous campaign that is almost completed

\begin{align*}
K^0_S K^0_L, & \ K^0_S K^0_L \pi^+ \pi^-, \ K^0_S K^0_S \pi^+ \pi^-, \ K^0_S K^0_S K^+ K^- \\
\bar{p}p & , \ K^+ K^- \\
2(\pi^+ \pi^-) & , \ K^+ K^- \pi^0, \ K^+ K^- \pi^+ \pi^- \\
\pi^+ \pi^- & , \ K^+ K^- \pi^+ \pi^-, \ K^+ K^- \pi^0 \pi^0 \\
K^+ K^- \eta, & \ K^+ K^- \pi^0, \ K^0 K^\pm \pi^\mp \\
2(\pi^+ \pi^-) \pi^0, & \ 2(\pi^+ \pi^-) \eta, \ K^+ K^- \pi^+ \pi^- \pi^0, \ K^+ K^- \pi^+ \pi^- \eta \\
K^+ K^- \pi^+ \pi^-, & \ K^+ K^- \pi^0 \pi^0, \ K^+ K^- K^+ K^- \\
\Lambda \bar{\Lambda}, & \ \Lambda \Sigma^0, \ \Sigma^0 \bar{\Sigma}^0 \\
3(\pi^+ \pi^-), & \ 2(\pi^+ \pi^-) \pi^0, \ K^+ K^- 2(\pi^+ \pi^-) \\
\bar{p}p & , \ K^+ K^- \pi^+ \pi^-, \ K^+ K^- K^+ K^- \\
\pi^+ \pi^- \pi^0 & 
\end{align*}

Preliminary

\begin{align*}
\text{Phys. Rev. D87 (2013) 092005} & \\
\text{Phys. Rev. D88 (2013) 032013} & \\
\text{Phys. Rev. D85 (2012) 112009} & \\
\text{Phys. Rev. D86 (2012) 012008} & \\
\text{Phys. Rev. D86 (2012) 032013} & \\
\text{Phys. Rev.D76 (2007) 092005} & \\
\text{Phys. Rev.D76 (2007) 012008} & \\
\text{Phys. Rev. D76 (2007) 092006} & \\
\text{Phys. Rev.D71 (2005) 052001} & \\
\end{align*}

First measurements, superseded, \(454 - 469 \text{ fb}^{-1}, 232 \text{ fb}^{-1}, 89 \text{ fb}^{-1}\)
Future $g$-2 Prospects

• Fermilab $g$-2 collaboration (E989), 0.14 ppm
  – Plans to reduce uncertainty by factor of 4
  – E821 magnet was moved from Brookhaven to Fermilab in July 2013

• J-PARC, 0.1 ppm
  – Use ultra-cold muon beam to limit transverse momentum
  – Eliminates electric focusing field, no “magic” momentum

• Theory improvements continue
  – Including use of more precise $e^+e^- \rightarrow$ hadrons cross-sections from *BABAR* using ISR (initial-state radiation) events
Summary

• **BABAR** has carried out an extensive program of ISR cross-section measurements
  – Have the most precise measurements of $e^+e^- \rightarrow$ hadrons from threshold to $E_{\text{cut}} = 1.8$ GeV and above

• Uncertainty on $a_\mu$
  – Standard model calculation: $6.0 \times 10^{-10}$
    • Dominated by systematic uncertainties on $\sigma(e^+e^- \rightarrow$ hadrons)
    • Watch for results from CMD-3 & SND 2000 @ VEPP-2000
  – Experiment BNL E821 measurement: $6.3 \times 10^{-10}$
  – Standard model vs. experiment still around $3.5 \sigma$ difference

• New experimental measurements to come from Fermilab E989 and J-PARC
  – 0.54 ppm uncertainty to be reduced to around 0.1 ppm
Extra Slides
The BABAR detector at PEP-II

- 1.5 T solenoid (superconducting)
- Calorimeter 6580 CsI(Tl) crystals
- Cherenkov Detector 144 fused silica bars 11,000 PMTs
- e⁺ (3.1 GeV)
- e⁻ (9 GeV)
- Silicon Vertex Tracker 5 double-sided layers
- Drift Chamber 40 layers
- Instrumented Flux Return 18–19 layers

NIM A479, 1 (2002); Update: NIM A729, 615 (2013)
Muon anomalous magnetic moment

- Lepton magnetic moment anomaly, sensitivity to new physics
- Experimental value
  - $a_\mu = 116592089 \pm 63 \times 10^{-11}$
    - BNL E821: Bennett et al., PRD 73, 072003 (2006); RPP 2013
  - $a_\mu^{SM} = 116591802 \pm 49 \times 10^{-11}$
  - Inconsistent at 3.6$\sigma$ level
- Standard Model calculation
- $a_\mu^{(QED)} = 116584718.10 \pm 0.15 \times 10^{-11}$
- $a_\mu^{(weak)} = 154 \pm 2 \times 10^{-11}$
- $a_\mu^{(had)} = 6930 \pm 49 \times 10^{-11}$
  - Engel et al., PRD 86 037502 (2012); Davier et al., EPJ C71 1515 (2011)
$e^+e^- \rightarrow p \text{ anti-}p$

$\gamma$ detected

$\gamma$ undetected


\( \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \)

- \( \pi \rightarrow \mu \nu \) violates P, \( \mu \) longitudinally polarized.
- \( \mu \)'s at "magic momentum" \( \approx 3.1 \text{ GeV/c} \), in a storage ring with constant \( \vec{B} \).
  - \( \mu \) rotating with freq \( \omega_c \); \( \mu \) spin precessing with freq \( \omega_s \)
  - freq. difference \( \omega_a = \omega_s - \omega_c = a_\mu eB/m_\mu \)
- \( \mu \rightarrow e \nu \bar{\nu} \) violates P, \( e \) direction (energy in lab) remembers \( \mu \) polarization.
  \( \Rightarrow \) Fraction of detected \( e \) above a \( E_{\text{threshold}} \) is modulated with freq. \( \omega_a \)

\[
a_\mu (\text{expt}) = (11659208.0 \pm 5.4(\text{stat}) \pm 3.3(\text{syst})) \times 10^{-10} \quad (0.54 \text{ ppm})
\]

E821 @ BNL, \( \mu^+ - \mu^- \) charge average Bennett Phys. Rev.D73 (2006) 072003

14 December 2013 Cowan — Measurement of e+e− → hadrons cross sections at BABAR, and implications for the muon g−2 Miami 2013
Why we believe QED is correct

• ae measured in a one-electron quantum cyclotron
• \( ae = 1159652180.73(\pm 0.28) \times 10^{-12}, \) (0.24 ppb)
• • ) known to 0.37 ppb
• • In total the QED uncertainty on aQED
• \( \mu \) is tiny : 1.7 ppb
Lepton anomalous magnetic moments

- Gyromagnetic factor \( g \)
  \[
  \vec{\mu} = g \frac{e}{2m} \vec{s}, \quad a = \frac{(g - 2)}{2}
  \]

- (1928) Pointlike Dirac particles: \( g = 2, \ a = 0. \)
  \( g \neq 2 \) due to higher order contributions

- (1947) Nafe measure
  \[ a_e = (2.6 \pm 0.5) \times 10^{-3} \]

- (1948) Schwinger (1st order in \( \alpha \))
  \[ a^{(1)} = \frac{\alpha}{2\pi} \approx 1.2 \times 10^{-3} \]

- Together with Tomonaga’s completion of renormalized calculation of Lamb shift,
  Basis of our belief in QED and in the gauge-theory-based SM
Abstract

• The BABAR Collaboration has made extensive measurements of cross sections of $e^+e^- \rightarrow$ hadrons for $KK$, $\pi\pi$, and four- and six-body hadronic decays. Cross sections are studied in the energy range from 1 to 3 GeV using initial-state radiation. These measurements allow improved precision of the predicted value of $(g-2)_\mu$, the muon anomalous magnetic moment.