High-Energy String Scattering and Large N QCD

Francisco Rojas
University of Florida
Miami Conference 2011

Based on: arXiv:1105.3967 (with C. Thorn)
Motivation

- Use of Open String Theory as a tool to understand non-perturbative aspects of QCD such as confinement.
- Gauge/Gravity duality approach to Large $N$ QCD
  - Necessary condition $R/l_s \gg 1$ (small curvature)
  - Large $N$ QCD possibly has a gravity dual
  - But probably $R/l_s \sim O(1)$ (cannot trust semiclassical approximation)
- From the early days of string theory it has been recognized that $\alpha' \rightarrow 0$ limit is a field theory.
- We want to address nonperturbative issues of Large $N$ QCD using this connection. Final goal:

\[ \sum \text{Large } N \text{ QCD diagrams} \]

- Too many diagrams
Strategy:

\[ \sum_{\text{loops}} \text{Planar Open Strings} \xrightarrow[\alpha' \to 0]{\Rightarrow} \sum_{\text{loops}} \text{Large N QCD diagrams} \]

- The L.H.S. might be more tractable than the R.H.S.
- What string theory to use?
  The choice is certainly not unique. Simplest?
  - Subcritical string in \( d = 4 \). (coupling to closed strings?) \( \times \)
  - Bosonic string (contains open string tachyons) \( \times \)
  - RNS, Superstrings. We do not want fermions \( \times \)
  - **Even G-parity sector of the Neveu-Schwarz model (NS+)** \( \checkmark \)

- With the open string ends attached to \( N \) coincident D3-branes the \( \alpha' \to 0 \) limit of this string model **is pure Yang-Mills in \( d = 4 \)**
• How to implement the sum?
• Mandelstam’s light-cone interacting picture for open strings:

<table>
<thead>
<tr>
<th>Light-cone picture</th>
<th>Lattice Light-cone picture</th>
</tr>
</thead>
</table>

• Planar graph summation for the Bosonic string [Giles & Thorn, 1977]
• Numerical simulations using Monte Carlo methods [Orland, 1986]
• Why not try this for the NS+ string? [Thorn, 2010]
• The lattice description for the NS+ string turns out to be described by a critical Ising model.
• Keeping the closed string tachyon causes instabilities.
• Resolution of this instability could explain confinement by pointing in the direction of the true confining vacuum of (large $N$) QCD.
Using the even G-parity sector of the NS string model (NS+) we compute the one-loop $M$-“gluon” amplitude with open strings attached to $N$ coincident D$p$-branes:

$$M = k \int_{0}^{1} \frac{dq}{q^2} \left( \frac{-1}{\log q} \right)^{(10-D)/2} f(q) \int_{R} d\nu_{M} \cdots d\nu_{2} \prod_{i<j}^{M} \psi(\nu_{ji})^{2\alpha' k_{i} \cdot k_{j}} \langle P_{1} \cdots P_{M} \rangle$$

where

$$\psi(\nu) = \frac{\theta_{1}(\nu, q)}{\theta'_{1}(0, q)} , \quad P_{i} = P_{i}(dn(\nu), sn(\nu), cn(\nu))$$

$$R : \quad 0 \leq \nu_{2} \leq \nu_{3} \leq \cdots \leq \nu_{M} \leq 1$$

$$f(q) : \quad \text{analytic function of } q$$

where we have omitted the group theory factor $\text{Tr}(\lambda_{1} \cdots \lambda_{M})$
There are spurious divergences in $\int \prod_k d\nu_k$ due to momentum conservation. Regulate them using $\sum_i k_i = p$ (Goddard, Neveu-Scherk '71)

$$\mathcal{M}(p) = [\mathcal{M}(p) - C(p)]_{p=0} + C(p)$$

Analytically continue to $p = 0 \rightarrow C(p) \propto \mathcal{M}_{\text{Tree}}$

There is a $q^{-2}$ divergence (UV) $\rightarrow$ absorbed in $g$ (Neveu, Scherk '71)

Would-be subleading divergence (dilaton emission into the vacuum) is absent for $D < 8$ due to the presence of D-branes:

$$\int_0^1 \frac{dq}{q} \rightarrow \int_0^1 \frac{dq}{q} \left(\frac{-\pi}{\ln q}\right)^{(10-D)/2}$$

Everything completely finite for $4 < D < 8$
4-gluon amplitude

Mandelstam’s variables:

\[ s = -(k_1 + k_2)^2 \]
\[ t = -(k_2 + k_3)^2 \]

Regge limit: \( \alpha' s \to -\infty \) with \( t \) fixed.

At tree level:

\[ A_4^{\text{tree}} \approx -2g^2 \epsilon_1 \cdot \epsilon_4 \epsilon_2 \cdot \epsilon_3 \left(-\alpha' s\right)^{\alpha(t)} \Gamma(-\alpha' t) \]

where \( \alpha(t) = 1 + \alpha' t \).

At \( t = 0 \), \( \alpha(0) = 1 \Rightarrow \) gluon is massless.

We want the one-loop correction:

\[ \alpha(t) = 1 + \alpha' t + \Sigma(t) \]

Will \( \Sigma(t) \) be zero at \( t = 0 \)?
Yes! \( \Rightarrow \) gluon remains massless at one-loop for this model. ✔
In the Regge limit the amplitude becomes

\[(\beta(t) + \delta \beta)s^{\alpha(t)+\delta \alpha} \approx \beta(t)s^{\alpha(t)} + \delta \alpha \beta(t)s^{\alpha(t)} \log s + \delta \beta(t)s^{\alpha(t)}\]

It is dominated by the region \(\nu_2 \sim \nu_3, \nu_4 \sim 1\). Extracting the contribution from that region yields (F.R., Thorn '11):

\[\mathcal{M}_4^\pm \sim -g^2 \epsilon_1 \cdot \epsilon_4 \epsilon_2 \cdot \epsilon_3 (-\alpha')^{1+\alpha' t} \Gamma(-\alpha' t) \log(-\alpha' s) \Sigma(t)\]

from which we identify \(\Sigma(t)\).

---

Open string Regge trajectory through one-loop

\[\alpha(t) = 1 + \alpha' t + \Sigma(t)\]

where

\[\Sigma(t) = C g^2 \int_0^1 \frac{dq}{q} \left(\frac{-\pi}{\ln q}\right)^{(10-D)/2} \int_0^{\pi} d\theta \left((-\psi^2 [\ln \psi]'')^{\alpha' t} \frac{\alpha' t}{[\ln \psi]''} (P_+ X^+ - P_- X^-) \right.\]

\[-\frac{1}{4} (P_+ - P_-) \left((-\psi^2(\theta)[\ln \psi]'')^{\alpha' t} - 1\right) \left(-\ln \psi\right)'\]
Compute $\Sigma(t = 0)$. The integral that defines $\Sigma(t)$ reduces to:

$$\int_0^1 dq \left( \frac{-\pi}{\ln q} \right)^{(10-D)/2} \int_0^\pi d\theta \left( [\log \psi]'' + \frac{1}{\sin^2 \theta} \right)$$

where

$$\psi \equiv \frac{\theta_1(\theta, q)}{\theta'(0, q)}$$

The integral over $\theta$ is:

$$\int_0^\pi d\theta \left( [\log \psi]'' + \frac{1}{\sin^2 \theta} \right) = \left( [\log \psi]' - \cot \theta \right) \bigg|_{\theta=0}^{\pi}$$

$$= \left( \sum_{n=1}^\infty \frac{4q^{2n} \sin 2\theta}{1 - 2q^{2n} \cos 2\theta + q^{4n}} \right) \bigg|_{\theta=0}^{\pi}$$

$$= 0$$

Thus, $\Sigma(t)$ formally vanishes at $t = 0 \therefore$ gluon stays massless at one-loop.

But, $t \rightarrow 0$ is the infrared limit which should reflect the reggeization of the gluon in Yang-Mills. Field theory calculations show that

$$\Sigma_{YM}(t) = -\frac{g^2}{4\pi^2} \left[ \frac{2}{D-4} + \ln(-\alpha' t) \right]$$

[Kunszt, Signer, Trocsanyu]
What about the integration over $\int dq$?

Indeed, the $q \sim 1$ region produces non-analytic behavior as $t \to 0$.

The $q \sim 1$ region is better analyzed performing the imaginary Jacobi transformation: $\theta_1(\nu|\tau) \to \theta_1(-\nu/\tau| - 1/\tau)$

where $q = \exp[2\pi^2/\log w]$ which maps $q \sim 1$ to $w \sim 0$.

The amplitude becomes

$$\Sigma(t) = C g^2 \int_0^1 \frac{dw}{2w} \left(\frac{-2\pi}{\ln w}\right)^{-3+D/2} \int_0^\pi d\theta \left((-\psi^2[\ln \psi]''')^{\alpha' t} \frac{\alpha' t}{[\ln \psi]''}(P_+ X^+ - P_- X^-) \right.$$

$$\left. - \frac{1}{4}(P_+ - P_-) \left((-\psi^2[\ln \psi]''')^{\alpha' t} - 1\right) [-\ln \psi]''\right)$$
The region \( w \sim 0 \) can now produce non-analytic behavior as \( t \to 0 \). This goes as

\[
\Sigma \approx -\int_0^{\delta} \frac{dw}{w} \left( \frac{-2\pi}{\ln w} \right)^{D/2-3} \int_0^{\pi} d\theta \left( (-\psi^2 [\ln \psi]''')^{\alpha' t} \frac{\alpha' t}{[\ln \psi]''} \right) \left[ 2 + \frac{D + S - 2}{2\pi^2} \left( \frac{2\pi}{\ln w} \right)^2 \right] \\
- \frac{D + S - 2}{8} \left( \frac{2\pi}{\ln w} \right)^4 \left( (-\psi^2 [\ln \psi]''')^{\alpha' t} - 1 \right) \left[ -\ln \psi'' \right)
\]

With the result

\[
\Sigma \approx -g^2 \frac{\Gamma(D/2 - 2)^2}{\Gamma(3 - D/2)} \right)\frac{(-\alpha' t)^{(D-4)/2}}{\Gamma(D - 4)}
\]

For \( D \to 4 \)

\[
\Sigma_{D\to 4} \approx -\frac{g^2}{4\pi^2} \left[ \frac{2}{D - 4} + \ln(-\alpha' t) \right]
\]

which now matches with the expected result from Yang-Mills!
Hard scattering limit

- Hard scattering limit: \( s \to -\infty \) with \( t/s \equiv -\lambda \) held fixed.
- At tree level

\[
\mathcal{M} \sim e^{-\alpha'|s|g(\lambda)}
\]

- We now wish to study this regime for the one-loop amplitude
- Recalling the one-loop amplitude

\[
\mathcal{M} = C g^4 \int_0^1 \frac{dq}{q^2} \left( \frac{-1}{\log q} \right)^{(10-D)/2} f(q) \int_R d\nu_4 \cdots d\nu_2 \prod_{i<j}^4 \psi(\nu_{ji})^{2\alpha'k_i \cdot k_j} \langle P_1 \cdots P_4 \rangle
\]

It is more convenient to write

\[
\psi(\nu_{ji})^{2\alpha'k_i \cdot k_j} \equiv e^{-\alpha' s V_\lambda}
\]

where

\[
V_\lambda = V(\nu_2, \nu_3, \nu_4, q)
\]

Thus, all we need are all the stationary points of \( V_\lambda \) in the integration region above. Then, we expand about these points.
When studying the renormalization of the original dual resonance models, Neveu and Scherk [1971] used the change of variables:

\[ r(\nu_3) = \frac{\sin \pi \nu_4}{\sin \pi \nu_3}, \quad x(\nu_2) = \frac{\sin \pi \nu_4 \sin \pi \nu_2}{\sin \pi \nu_4 \sin \pi \nu_3} \]

In this new variable we have

\[ V_\lambda = \ln x - \lambda \ln(1 - x) + 2 \sum_{n=1}^{\infty} \frac{1}{n} \frac{q^{2n}}{1 - q^{2n}} (S_n - \lambda T_n) \]

which has many stationary points that are not exponentially suppressed.

The dominant one turns out to be

\[ x = (1 - \lambda)^{-1}, \quad q = 0 \]

Now we expand the integrand about these regions.
We obtained

\[ \mathcal{M} \sim F(\lambda) \, e^{-\alpha' |s| g(\lambda)} \, [\log(-\alpha' s)]^{1-p} \]

with

\[
F(\lambda) = (-\alpha' s)^2 \int_0^\pi d\theta \int_0^{\infty} dr \frac{r^2 \sin^2 \theta}{(r^2 + 2r \cos \theta + 1)(r^2(1-\lambda)^2 + 2r(1-\lambda) \cos \theta + 1)}
\]

Compare this result with the Regge behavior by taking \( s \gg t \), i.e.: \( \lambda \sim 0 \).
As \( \lambda \to 0 \), \( F(\lambda) \) is dominated by the region \( r \sim 1, \, \theta \sim \pi \)

\[
F(\lambda) \sim 2(-\alpha' s)^2 \log(-\alpha' s)
\]

thus

\[
\mathcal{M} \sim e^{-\alpha' |s| g(\lambda)} \, (-\alpha' s)^2 \, [\log(-\alpha' s)]^{2-p}
\]

This should match with the Regge behavior at high \( \alpha' t \).
Recall that in the Regge limit
\[ \mathcal{M}_4^\pm \sim -g^2(-\alpha's)^{1+\alpha't} \Gamma(-\alpha't) \log(-\alpha's) \Sigma(t) \]

We need \( \alpha't \gg 1 \) above:

- Stirling’s approx.: \( \Gamma(-\alpha't) \sim \sqrt{2\pi}(-\alpha't)^{-1/2-\alpha't} e^{\alpha't} \)
- \( \Sigma(t) \) for small \( t \) is dominated by the \( q \sim 0 \) region in

\[
\Sigma(t) = Cg^2 \int_0^1 dq \left( \frac{-\pi}{\ln q} \right)^{(10-D)/2} \int_0^\pi d\theta \left( -\psi^2[\ln \psi]'' \right)^{\alpha't} \frac{\alpha't}{[\ln \psi]''} (P_+X^+ - P_-X^-) \\
- \frac{1}{4}(P_+ - P_-) \left[ (-\psi^2(\theta)[\ln \psi]''\right)^{\alpha't} - 1] [-\ln \psi]'' \right)
\]

- Extracting the dominant contribution we obtain

\[
\Sigma(t) \sim \alpha't [\log(-\alpha't)]^{1-p}
\]

- Combining everything yields (\( \alpha'|t| \gg 1 \) but still \( |t| \ll |s| \))

\[
\mathcal{M} \sim e^{-\alpha'|s|g(\lambda)} (-\alpha's)^2 [\log(-\alpha's)]^{2-p}
\]

which is the desired result.
Conclusions

- We studied another aspect about the connection: **Open Strings - Large \( N \) QCD**

- One-loop correction of planar open string Regge trajectory:
  \[
  \alpha(t) = 1 + \alpha't + \Sigma(t)
  \]

- Field theory limit: \( \Sigma(t)_{\text{string}} \approx C(-\alpha't)^{(D-4)/4} \)

- \( \Sigma_{D \to 4} \approx -\frac{g^2}{4\pi^2} \left[ \frac{2}{D-4} + \ln(-\alpha't) \right] \)

  \[ \to \text{same as dimensionally regularized Yang-Mills} \]

- Hard scattering regime: Similar to tree level exponential suppression, although by powers of \( s \)

  \[
  \mathcal{M} \sim e^{-\alpha'|s|g(\lambda)} (-\alpha's)^2 [\log(-\alpha's)]^{2-p}
  \]

- **Calculational check:** Regge limit matches with hard scattering regime as \( \alpha'|t| \gg 1 \)
On more speculative grounds:
- We remind ourselves that our final goal is to understand confinement in large $N$ QCD.
- Main tool:

\[ \sum_{\text{loops}} \text{Planar open strings diagrams} \]

- Keeping closed string tachyon: resolution of the instability induced by its presence hopefully teaches us something about the true confining vacuum.
- They studied the planar diagrams in the Regge limit at all loops.
- They were able to perform the sum of all leading terms

\[ \sum_{h=1}^{\infty} A_h(s, t) \quad h: \text{number of hole insertions} \]

- The number of holes $h$ correspond to the number of loops in the dual open string picture (open/closed string duality).
- The leading terms all come from the $q \sim 0$ region at one-loop.
- This is precisely the region where the closed string tachyon dominates in the dual open string picture (hard scattering).
- Can we perform the same sum in our NS+ model? (work in progress)