Ground-state baryon decuplet form factors

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Introduction and Abstract

The properties of the ground-state spin $\frac{3}{2}$ baryon decuplet magnetic moments $\Delta^-, \Xi^*, \Sigma^*$ and $\Omega^-$ and their ground-state spin $\frac{1}{2}$ cousins $p, n, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^+, \Xi^-$ have been studied for many years with a modicum of success. The magnetic moments of many are yet to be determined [1]. Of the decuplet baryons, only the magnetic moment of the $\Omega^-$ has been accurately determined. In this talk, we study the electric charge and magnetic dipole *physical* decuplet form factors as a function of 4-momentum squared without ascribing any specific form to quark structure or intra-quark interactions.
• While the masses (pole or otherwise) and other physical observables of some of these particles have been ascertained, the magnetic moments of many are yet to be determined.

• For the spin $\frac{3}{2}$ baryon decuplet, the experimental situation is poor—only the magnetic moment of the $\Omega^-$ has been accurately determined. Nothing is known about form factor behavior.

• The reasons for this paucity of data for the decuplet particle members are the very short lifetimes due to available strong interaction decay channels and the existence of nearby particles with quantum numbers which allow for configuration mixing.

• The $\Omega^-$ is an exception in that it is composed of three valence $s$ quarks which make its lifetime substantially longer than any of its decuplet partners which have many more decay channels available.
Many theoretical models have been put forth over the past few decades. In addition to the simplest $SU(3)$ model, early seminal ones are the $SU(6)$-like models put forth by Bég, Lee, and Pais [2], Gerasimov [3], Gürsey and Radicati [4], Sakita [5], and others.

An excellent source of information is the book by Lichtenberg [6].

Typically, these models invoke the additivity hypothesis where a hadron magnetic moment is given by the sum of its constituent quark magnetic moments.
• A number of theoretical and computational investigations involving the magnetic moments of the \( \Omega^- \) and the \( \Delta^- \) and lattice QCD (quenched and unquenched, unphysical pion mass) techniques have been used and show promise [7,8,9].

• More recently, a non-perturbative approach was used where the magnetic moments of the \textit{physical} decuplet \( \Upsilon \text{ spin } = \frac{3}{2} \) quartet members were calculated without ascribing any specific form to their quark structure or intra-quark interactions [10].

• A review which focuses on some theoretical and experimental approaches to the study of specific processes involving the \( \Delta(1232) \) can be found in Ref. [11].
The infinite momentum frame—in conjunction with the fact that the 4-vector electromagnetic current $j_{em}^{\mu}$ obeys the equal time commutator $[V_K^{0}, j_{em}^{\mu}] = 0$ even in the presence of symmetry breaking—can be used to calculate the magnetic moments of the physical decuplet $U$ Spin = $\frac{3}{2}$ quartet members without ascribing any specific form to their quark structure or intra-quark interactions [11,12,13,14,15]. That calculation requires knowledge of the electric charge and magnetic dipole form factors when the 4-momentum squared ($q_B^2 \equiv -Q_B^2$) is zero.

We next very briefly review the use of equal-time commutation relations (ETCRs) in the infinite momentum frame which provide the formulism for our study.
ETCRs in the Infinite Momentum Frame

- Equal-time commutation relations (ETCRs) used involve at most one current density, thus, problems associated with Schwinger terms are avoided;

- ETCRs involve the vector and axial-vector charge generators (the $V_\alpha$ and $A_\alpha$ \{\(\alpha = \pi, K, D, F, B, \ldots\)\}) of the symmetry groups of quantum chromodynamics (QCD);

- They are valid even though these symmetries are broken [15,16,17,18]

  and even when the Lagrangian is not known or cannot be constructed.
Infinite Momentum Frame Asymptotic $SU_F(N)$ Symmetry

- The annihilation operator $a_\alpha(\vec{k}, \lambda)$ of a physical on-mass-shell hadron maintains its linearity (including $SU_F(N)$ particle mixings) under flavor transformations generated by the charge $V_\alpha$ but only in the limit $|\vec{k}| \to \infty$

$$
\left[ V_i, a_\alpha(\vec{k}, \lambda) \right] = i \sum_\beta u_{i\alpha\beta}(\vec{k}, \lambda) a_\beta(\vec{k}, \lambda) + \delta u_{i\alpha\lambda}(\vec{k});
$$

\hspace{1cm} (1)

- The \textit{dynamical} assumption of asymptotic $SU_F(N)$ symmetry then states that

$$
\delta u_{i\alpha\lambda}(\vec{k}) \to (|\vec{k}|)^{-(1+\epsilon)}, \ (\epsilon > 0) \ \text{for} \ |\vec{k}| \to \infty
$$

\hspace{1cm} (2)
\[ |\alpha, \vec{k}, \lambda \rangle = \sum_j C_{\alpha j} |j, \vec{k}, \lambda \rangle, \vec{k} \to \infty; \quad (3) \]

- The orthogonal matrix \( C_{\alpha j} \) depends on \( SU_F(N) \) mixing parameters and can be constrained directly by the ETCRs without introducing an ad hoc mass-mixing angle matrix;

- All nonlinear terms vanish like \( |\vec{k}|^{-(1+\epsilon)} \), \( \epsilon > 0 \), as \( \vec{k} \to \infty \). In the \( \infty \)-momentum frame, the physical annihilation operator \( a_\alpha(\vec{k}, \lambda) \) is related linearly to the representation annihilation operator \( a_j(\vec{k}, \lambda) \) via the orthogonal mixing matrix \( C_{\alpha j}(\lambda) \);

- Even in severely broken \( SU_F(N) \) symmetry—such as \( SU_F(4) \) or \( SU_F(5) \)—asymptotic \( SU_F(N) \)-symmetry methods are applicable.
The Rarita-Schwinger Spinor

For the on-mass shell $J^P = 3/2^+$ ground state decuplet baryon B with mass $m_B$, the Lorentz- covariant and gauge-invariant electromagnetic current matrix element in momentum space where the 4-momentum vectors $P \equiv p_1 + p_2$, $q \equiv p_2 - p_1$ and $\lambda_1$ and $\lambda_2$ represent helicity is given by:

$$
\frac{e}{(2\pi)^3} \sqrt{\frac{m_B^2}{E_B^l E_B^s}} \times \bar{u}^\alpha_B(p_2, \lambda_2) \left[ \Gamma_{\alpha\beta}^{\mu} \right] u^\beta_B(p_1, \lambda_1),
$$

(4)

$$
\Gamma_{\alpha\beta}^{\mu} = g_{\alpha\beta} \left\{ (F_1^B(q_B^2) + F_2^B(q_B^2)) \gamma^\mu - \frac{P^\mu}{2m_B} F_2^B(q_B^2) \right\} + \frac{q_\alpha q_\beta}{(2m_B)^2} \left\{ (F_3^B(q_B^2) + F_4^B(q_B^2)) \gamma^\mu - \frac{P^\mu}{2m_B} F_4^B(q_B^2) \right\},
$$

(5)
\[ e = +\sqrt{4\pi\alpha}, \alpha = \text{the fine structure constant}, \text{the } F_i^B \]

are the four \( \gamma^* BB \) form factors and \( \Gamma_{\alpha\beta}^{\mu} \) is written in a very useful form using the Gordon identities, \( Q_B = \text{charge of baryon } B \text{ in units of } e, \) and \( \mu_B = \text{the magnetic moment (measured in nuclear magneton units } \mu_N = e/(2m), m = \text{proton mass} \) of baryon \( B \) with:

\[
\begin{align*}
F_1^B(q_B^2) & \sim \text{electric charge form factor} \\
F_2^B(q_B^2) & \sim \text{magnetic dipole form factor} \\
F_3^B(q_B^2) & \sim \text{electric quadrupole moment form factor} \\
F_4^B(q_B^2) & \sim \text{magnetic octupole form factor}
\end{align*}
\]

\[ Q_B = F_1^B(0) e, \tag{6} \]
\[ \mu_B = \left[ (F_1^B(0) + F_2^B(0))\left(\frac{m}{m_B}\right) \right] \mu_N. \tag{7} \]
The baryon Rarita-Schwinger spinor \([18]\) \(u_B^\sigma(v_B, \theta, \lambda)\)
with helicity \(\lambda\), angle \(\theta\) referred to the \(z\)-axis established by \(\vec{t} = |\vec{t}| \hat{z}\), and with velocity parameter \(v_B\) is given by:

\[ u^\mu_B(v_B, \theta, \lambda) = \]
\[ \sum_{m_1=-\frac{1}{2}}^{+\frac{1}{2}} \sum_{m_2=-\frac{1}{2}}^{+\frac{1}{2}} \langle \frac{1}{2}, \frac{1}{2} | m_1, m_2, \lambda \rangle u_B(v_B, \theta, m_1) \epsilon^\mu_B(v_B, \theta, m_2). \]
\[ (8) \]

\[ u_B(v_B, \theta, m_1) = \]
\[ \begin{pmatrix}
\cosh(\frac{v_B}{2}) \cos(\theta) \delta_{m_1, \frac{1}{2}} - \sin(\frac{\theta}{2}) \delta_{m_1, -\frac{1}{2}} \\
\cosh(\frac{v_B}{2}) \sin(\theta) \delta_{m_1, \frac{1}{2}} + \cos(\frac{\theta}{2}) \delta_{m_1, -\frac{1}{2}} \\
\sinh(\frac{v_B}{2}) \cos(\theta) \delta_{m_1, \frac{1}{2}} + \sin(\frac{\theta}{2}) \delta_{m_1, -\frac{1}{2}} \\
\sinh(\frac{v_B}{2}) \sin(\theta) \delta_{m_1, \frac{1}{2}} - \cos(\frac{\theta}{2}) \delta_{m_1, -\frac{1}{2}} 
\end{pmatrix}, \]
\[ (9) \]
\[ \epsilon_{B}^{\lambda}(v_{B}, \theta, m_{2}) = \begin{pmatrix}
\sinh(v_{B}) \delta_{m_{2}}, 0 \\
-\frac{m_{2}^{2}}{\sqrt{2}} \cos(\theta) \delta_{|m_{2}|}, 1 + \cosh(v_{B}) \sin(\theta) \delta_{m_{2}}, 0 \\
-\frac{i}{\sqrt{2}} \delta_{|m_{2}|}, 1 \\
\frac{m_{2}^{2}}{\sqrt{2}} \sin(\theta) \delta_{|m_{2}|}, 1 + \cosh(v_{B}) \cos(\theta) \delta_{m_{2}}, 0
\end{pmatrix}. \]

\[ \epsilon_{B}^{\lambda}(v_{B}, \theta, m_{2}) \] is the baryon polarization \((m_{2})\) 4-vector, \(u_{B}(v_{B}, \theta, m_{1})\) is a Dirac spinor with helicity index \(m_{1}\), and \(\langle 1/2, 1, 3/2|m_{1}, m_{2}, \lambda \rangle\) is a Clebsch-Gordan coefficient where our conventions are those of Rose [19].
• Physical states are normalized with \( \langle \vec{p}' | \vec{p} \rangle = \delta^3(\vec{p}' - \vec{p}) \) and Dirac spinors are normalized by \( \bar{u}^{(r)}(p)u^{(s)}(p) = \delta_{r,s} \).

• Our conventions for Dirac matrices are \( \{ \gamma^\mu, \gamma^v \} = 2g^{\mu v} \) with \( \gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \), where \( g^{\mu v} = \text{Diag} (1, -1, -1, -1) \).

• The Ricci-Levi-Civita tensor is defined by \( \varepsilon^{0123} = -\varepsilon^{0123} = 1 = \varepsilon_{123} \). As usual, we use natural units where \( \hbar = c = 1 \) [20].
Momenta are defined by:

\[ p_1^\sigma = t^\sigma = \begin{pmatrix} m_B \cosh(\alpha_B) \\ 0 \\ 0 \\ m_B \sinh(\alpha_B) \end{pmatrix} = \begin{pmatrix} E_B^t \\ E_B^t \\ E_B^t \\ t \end{pmatrix}, \quad (11) \]

\[ p_2^\sigma = s^\sigma = \begin{pmatrix} m_B \cosh(\beta_B) \\ m_B \sin(\theta) \sinh(\beta_B) \\ 0 \\ m_B \cos(\theta) \sinh(\beta_B) \end{pmatrix} = \begin{pmatrix} E_B^s \\ E_B^s \\ E_B^s \\ s \end{pmatrix}. \quad (12) \]

In addition to obeying the Dirac equation—thus making the Gordon identities very useful—the Rarita-Schwinger spinors satisfy the subsidiary conditions \( \gamma_\mu u_B^\mu(p, \lambda) = p_\mu u_B^\mu(p, \lambda) = 0. \)
The $\Omega^-$ and the $\Delta^-$ Form Factor Relationship

We utilize the commutator $[V_{K^0}^{\mu}, j_{em}^\mu(0)] = 0$ in the infinite momentum frame inserted between the baryon pairs $(\Omega^- s^\sigma | \Xi^*^- t^\sigma), (\Xi^*^- s^\sigma | \Sigma^*^- t^\sigma), (\Sigma^*^- s^\sigma | \Delta^- t^\sigma)$ where each baryon has helicity $+3/2$ and $t_z \to \infty$ and $s_z \to \infty$. The internal intermediate states saturating the commutator belong to the ground state decuplet baryons with helicity $3/2$ which has the effect of restricting greatly the number of possible configuration mixing contributions coming from 56 or spin 3/2 members of 70 excited states and other low-lying supermultiplets. We obtain the following equation:
\[
\left\langle \Xi^* s^\sigma \right| V_{K^0} \left| \Omega^- s^\sigma \right\rangle \left\langle \Omega^- s^\sigma \right| j_{em}^{\mu} \left| \Omega^- t^\sigma \right\rangle \\
- \left\langle \Xi^* s^\sigma \right| j_{em}^{\mu} \left| \Xi^* t^\sigma \right\rangle \left\langle \Xi^* t^\sigma \right| V_{K^0} \left| \Omega^- t^\sigma \right\rangle = 0,
\]
\[
\left\langle \Sigma^* s^\sigma \right| V_{K^0} \left| \Xi^* s^\sigma \right\rangle \left\langle \Xi^* s^\sigma \right| j_{em}^{\mu} \left| \Xi^* t^\sigma \right\rangle \\
- \left\langle \Sigma^* s^\sigma \right| j_{em}^{\mu} \left| \Sigma^* t^\sigma \right\rangle \left\langle \Sigma^* t^\sigma \right| V_{K^0} \left| \Xi^- t^\sigma \right\rangle = 0,
\]
\[
\left\langle \Delta^- s^\sigma \right| V_{K^0} \left| \Sigma^* s^\sigma \right\rangle \left\langle \Sigma^* s^\sigma \right| j_{em}^{\mu} \left| \Sigma^* t^\sigma \right\rangle \\
- \left\langle \Delta^- s^\sigma \right| j_{em}^{\mu} \left| \Delta^- t^\sigma \right\rangle \left\langle \Delta^- t^\sigma \right| V_{K^0} \left| \Sigma^* t^\sigma \right\rangle = 0.
\]
(13)
Eq. (1), Eq. (2), and Eq. (3) in conjunction with Eq. (13) imply that:

$$
\langle \Delta^{-} s^\sigma, \lambda | j_{em}^\mu(0) | \Delta^{-} t^\sigma, \lambda \rangle = \langle \Omega^{-} s^\sigma, \lambda | j_{em}^\mu(0) | \Omega^{-} t^\sigma, \lambda \rangle,
$$

where $t_z \to \infty$, $s_z \to \infty$, and $\lambda = \text{helicity} = +3/2$

(14)

While Eq. (14) is reminiscent of what one obtains in pure unbroken $SU_F(N)$ symmetry with a $U$-spin singlet electromagnetic current, it is now obtained in broken symmetry. With $r$ (constant) $\geq 1$ ensuring no helicity reversal, one can explicitly evaluate Eq. (14) with $\mu = 0$ and $s_x = \theta = 0$ (collinear case) using Eq. (4)–Eq. (12) as was done in Ref. [10]. There we obtained:

$$
\lim_{t_z \to +\infty} \lim_{s_z \to +\infty} \left\{ \frac{1}{2} \cosh \left[ \frac{\alpha_{\Delta^{-}} - \beta_{\Delta^{-}}}{2} \right] \right\} \times (2 F_1^{\Delta^{-}} (q_{\Delta^{-}}^2) + F_2^{\Delta^{-}} (q_{\Delta^{-}}^2) - F_2^{\Delta^{-}} (q_{\Delta^{-}}^2) \cosh[\alpha_{\Delta^{-}} + \beta_{\Delta^{-}}]) =
$$

$$
\lim_{t_z \to +\infty} \lim_{s_z \to +\infty} \left\{ \frac{1}{2} \cosh \left[ \frac{\alpha_{\Omega^{-}} - \beta_{\Omega^{-}}}{2} \right] \right\} \times (2 F_1^{\Omega^{-}} (q_{\Omega^{-}}^2) + F_2^{\Omega^{-}} (q_{\Omega^{-}}^2) - F_2^{\Omega^{-}} (q_{\Omega^{-}}^2) \cosh[\alpha_{\Omega^{-}} + \beta_{\Omega^{-}}]).
$$

(15)
Upon taking the limits in Eq. (15) with $s_z = r t_z$ [r (constant) $\geq 1$ and $s_x = 0$] yields:

\[ F_{\Delta^-}^{\Delta^-} (q_{\Delta^-}^2) = \frac{m_{\Delta^-}^2}{m_{\Omega^-}^2} F_{\Delta^+}^{\Omega^-} (q_{\Omega^-}^2). \] (16)

In deriving Eq. (16), we utilized that, in general, even though $|\vec{s}|$ and $|\vec{t}| \to +\infty$, $q_B^2$ is finite and

\[ q_B^2 = -(1-r)^2 \frac{m_B^2}{r} - \frac{s_x^2}{r} = -Q_B^2. \]

\[ q_B^2 \big|_{s_x=0} = -(1-r)^2 \frac{m_B^2}{r}, \] (17)

\[ \cosh \left[ \frac{\alpha_B - \beta_B}{2} \right] \to \frac{1 + r}{2 \sqrt{r}}, \]
\[ \cosh [\alpha_B + \beta_B] \to \frac{2 r t_z^2}{m_B^2}, \] (18)
If we set \( r = 1 \Rightarrow q_{\Delta -}^2 = q_{\Omega -}^2 = 0 \), we obtain:

\[
F_1^{\Delta -}(0) = F_1^{\Omega -}(0) = -1
\]
\[
F_2^{\Delta -}(0) = \frac{m^2_{\Delta -}}{m^2_{\Omega -}} F_2^{\Omega -}(0).
\]  \hfill (19)

\[
\mu_{\Delta -} = \left[ (F_1^{\Omega -}(0) + \frac{m^2_{\Delta -}}{m^2_{\Omega -}} F_2^{\Omega -}(0)) \frac{m}{m_{\Omega -}} \right] \mu_N
\]
\[
= \left[ -(1 - \frac{m^2_{\Delta -}}{m^2_{\Omega -}} F_2^{\Omega -}(0)) \frac{m}{m_{\Omega -}} \right] \mu_N. \]  \hfill (20)
To obtain the relationship between the electric charge form factors, one again explicitly evaluates Eq. (14) but with $s_x \geq 0$ (constant and fixed), $0 \leq \theta \leq \frac{\pi}{2}$ (non-collinear case), and $r$ (constant) $\geq 1$ ensuring no helicity reversal. Analogously, We obtain:
• \[ F_{\frac{\Lambda^-}{1}}^{\Delta^-}(q^2_{\Delta^-}) = F_{\frac{\Omega^-}{1}}^{\Omega^-}(q^2_{\Omega^-}). \] (21)

• In deriving Eq. (21), we again utilized that, in general, even though \(|\vec{s}|\) and \(|\vec{t}|\) → +∞, \(q^2_B\) is finite and \(q^2_B = -\frac{(1-r)^2}{r}m^2_B - \frac{s^2_x}{r} \equiv -Q^2_B.\) (21)

Similar relations for the \(\Xi^{*-}\) and the \(\Sigma^{*-}\) can also be derived.
Conclusions

We have calculated the electric charge and magnetic dipole form factors of the ground state physical decuplet $U$ Spin $= \frac{3}{2}$ baryon quartet members $\Delta^-$, $\Sigma^{*-}$, and $\Xi^{*-}$ in terms of the $\Omega^-$ form factors without ascribing any specific form to their quark structure or intraquark interactions or assuming $SU_F(N)$ broken or unbroken symmetry or assuming an effective lagrangian.
References


[16] M. Gell-Mann, Physics 1, 63 (1964);


