Relativistic Thermodynamics

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“90 000 papers on “Black Holes”

3 papers from UCLA
Facts

Observation of “planets” in very small orbits.

There is a heavy object near the center of our galaxy:
Several million solar masses

By Newtonian model, the “planets” are at a safe distance:

600 times the Schwartzschild radius determined by the local gravitational field.

Andrea Ghez and about 15 collaborators:

Astrophysical J. Vol 620, 744-757 2005

do. Vol 622, 878-891, 2005

Interpretation

Black Hole: Name adopted by elimination of alternatives:

Convenient name for this object.

No reasonable physical model known.
No direct evidence of a horizon - how could there be?

No scenario of collapse can lead to

a Schwarzschild black hole with horizon.

Nothing passes the Schwarzschild horizon
Goal

This work is aimed at finding a model of a stellar object of the observed mass and size,

... allowing for an equation of state that can only apply to dark matter.

Strategy: Start from the outside, fixing the mass.

J. Hartle, Physics Reports Vol 46, 201-247.
Method

In order to say, with some authority, that there is an upper limit on the mass of a stellar object, it is best to have a class of theoretical models with the greatest possible number of reasonable theoretical features.
We shall need, in particular, to incorporate

General Relativity

and

Thermodynamics (including hydrodynamics).

This is my subject.
Unification of Gravitation and Thermodynamics

Scope and limitation of the project:

Divisions of Thermodynamics:

Equilibrium Thermodynamics

Adiabatic Thermodynamics

Extended Thermodynamics

(Relaxation, turbulence etc.)
Main idea: ACTION PRINCIPLE

Precursors:
Taub 1954
Bardeen 1970
Schultz 1970
Stage 1, the hamiltonian

Equilibrium thermodynamics, Gibbs 1875.

Extremize the “Hamiltonian”:

\[ F(V, T) + ST + VP, \]

Variational equations:

\[ \frac{\partial F}{\partial T} + S = 0, \quad \frac{\partial F}{\partial V} + P = 0. \]

Adiabatic equilibrium: S and P are fixed a priori.

System defined by the expression for the free energy F(V, T).
Stage 2, localization

(“extended thermodynamics”)

Hamiltonian density

\[ h = f(\rho, T) + sT + P, \]

Local variation

\[ \frac{\partial f}{\partial T} + s = 0, \quad \rho \frac{\partial f}{\partial \rho} - f = p. \]
Stage 3, hydrodynamics

The kinetic action should allow to include hydrodynamics, with its main axioms:

Equation of continuity
Bernoulli’s (Newton’s) equation.

It was discovered for one-component system by Fetter and Walecka.

\[ \mathcal{L} = \rho (\dot{\Phi} - \vec{v}^2/2 - \phi + \lambda) - f(\rho, T) - sT - P. \]

\[ \vec{v} = -\text{grad} \Phi. \]
Stage 4, General Relativity

\[ G_{\mu \nu} = T_{\mu \nu} \iff \text{Matter} \]

Tolman’s formula

\[ T_{\mu \nu} = (p + \rho)U_{\mu}U_{\nu} - p g_{\mu \nu} \]

is not enough. Especially for systems with several degrees of freedom.

Replacing it with an action principle is not difficult.

Challenge to astrophysicists: Onsager symmetry.
Non relativistic:

\[ \mathcal{L} = \rho(\dot{\Phi} - \vec{v}^2/2 - \phi + \lambda) - f(\rho, T) - sT + \frac{a}{3}T^4 - P. \]

Relativistic:

\[ \mathcal{L} = \rho(g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - c^2) - f(\rho, T) - sT + \frac{a}{3}T^4 - P. \]

To compare:

\[ \psi = c^2t + \Phi, \quad c \to \infty. \]

Critical question: Is this approach (relativistic or not),

possible,

convenient,

powerful,

innovative?

Claim: It helps me understand thermodynamics.
Example

Adiabatic lagrangian for an ideal gas

\[ \mathcal{L} = \rho (\dot{\Phi} - \vec{v}^2/2 - \phi + \lambda) - \mathcal{R} T \rho \log \frac{\rho}{T^n k_0}, \]

rho = density,

\( \Phi \) = velocity potential, \( \vec{v} = -\text{grad } \Phi \),

T = temperature,

n = adiabatic index,

\( k_0 \) = constant (entropy = \( \mathcal{R} \rho \ln k_0 \)),

\( \lambda \) = Lagrange multiplier (chemical potential),

\( \phi \) = gravitational potential.
Again: Lagrangian for an ideal gas:

\[ \mathcal{L} = \rho (\dot{\Phi} - \vec{v}^2/2 - \phi + \lambda) - \mathcal{R}T\rho \log \frac{\rho}{T^n k_0}. \]

This simple expression contains the following information.

1. The equation of continuity (variation of \( \Phi \)),
2. Bernoulli’s equation of motion (variation of \( \rho \)),
3. The gas law: \( p = \mathcal{R}T\rho \) (variation of \( \rho \))
4. The polytropic relations (variation of \( T \)),
5. The constant lapse rate of atmospheres,
6. The formula for the internal energy of an ideal gas,
7. The entropy, the free energy and the Gibbs free energy.
Stage 5, radiation

Stefan-Boltzmann term added:

\[ \mathcal{L} = \rho (\dot{\Phi} - \vec{v}^2/2 - \phi + \lambda) + \mathcal{R}T \log \frac{\rho}{T^n k_0} + \frac{a}{3} T^4, \]

The Stefan-Boltzmann term modifies the effective adiabatic index at very high temperatures, moving it asymptotically towards \( n = 3 \). (Compare Eddington, Chandrasekhar.)

\[ u = n \mathcal{R}T \rho + aT^4, \quad p = \mathcal{R}T \rho + \frac{a}{3} T^4 \]
Stage 6, generalizations

Heterogeneous systems.

\[ \mathcal{L} = \sum \rho_i (\dot{\Phi}_i - \bar{v}_i^2/2 - \phi + \lambda_i) - R_i T \rho_i \log \frac{\rho_i}{T n_i k_{0i}} \]

Note additivity of lagrangian. (Gibbs-Dalton hypothesis)

Two entropy parameters; how are they determined; how do they change when heat is added? Some answers:

(1) Molecular dissociation. The difference

\[ R_1 \ln k_{01} - R_2 \ln k_{02} \]

of specific entropies remains constant.

Variation leads directly to the law of mass action and Saha’s equation.
(2) Condensation (phase change, for example, using van der Waals free energy). The specific heat of evaporation is

$$\epsilon = T(\mathcal{R}_1 \ln k_0 - \mathcal{R}_2 \ln k_0).$$

This is Maxwell’s rule.
(3) Propagation of sound in mixed gases. Adiabatic lagrangian predicts two speeds in a 2-component mixture. Usual approach involves mechanical equations of motion and strong damping, even 2 temperatures.

Just adding an interaction term

$$\alpha \sqrt{\rho_1 \rho_2} \text{ or } \alpha T \sqrt{\rho_1 \rho_2}$$

leads to exactly the same formula for the speed of sound as a function of concentrations.

Apply to the propagation of adiabatic disturbances in the sun.

4. (To do.) Determination of critical points for mixtures.
In the works

A first 2-phase lagrangian for a hydrogen gas sphere with radiation - work in progress with Tom Wilcox.

(a) At low temperatures, in outer reason, there is hydrogen snow (alternatively, dark matter), with adiabatic index $> 5$.

(b) At some temperature there is a phase transition to hydrogen gas, with adiabatic index about $3/2$.

(c) At very low temperatures the Stefan-Boltzmann term drives the effective adiabatic index DOWN to 3. At very high high temperatures the Stefan-Boltzmann term drives $n$ UP to 3.
Preliminary conclusions:

There appears to be no limit on the mass.

Limits on temperature, density and pressure not yet known.
Summary

A formulation of thermodynamics as an action principle is conducive to insight and rigour.

The same lagrangian has to serve for all applications.

A relativistic version is very simple and direct, and convenient for calculations.

Preliminary result about super massive stars (not newsworthy, but all we have at this time): Stars can have very high mass, but in the model examined so far temperature and pressure become extremely high. The density falls off as the inverse cube of the distance.
Thank you.