Stringy Corrections, SUSY Breaking and the Stabilization of (all) Kähler moduli

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Overview

• Study the role of string corrections in type IIB flux compactifications

• \( \alpha' \) corrections to Kähler potential \( K \) are important even in large volume limit

• Motivation:
  • Stabilization of (all) Kähler moduli
  • Spontaneous supersymmetry breaking by F-terms
  • Construction of deSitter vacua (in addition to non-susy AdS and Minkowski)
Outline

• Review of flux compactifications
• Supersymmetric vacua and ample divisors
• Stringy corrections and supersymmetry breaking
• Implications
• Discussion and Outlook
Flux Compactifications (I)

We are interested in type IIB orientifold compactifications on Calabi-Yau three-fold $M$ (or equivalently F-theory on four-fold $X$) with non-trivial NS and R-R fluxes.

- Classical superpotential

$$W_0 = \int_M \Omega \wedge G$$

$$G = F - \tau H$$

- Tadpole condition

$$\frac{\chi(X)}{24} = \int_M F \wedge H$$

$\Omega$ is the holomorphic 3-form on $M$

$F$ and $H$ are the R-R and NS 3-form fluxes, respectively, through 3-cycles on $M$
Flux Compactifications (II)

The resulting N=1 supergravity scalar potential takes the standard form

\[ V = e^K (G^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2) \]

where \( D_i W = \partial_i W + W \partial_i K \quad G_{i\bar{j}} = \partial_i \partial_{\bar{j}} K \)

\[ K = -2 \log(\int_M J^3) - \log(\tau - \bar{\tau}) - \log(\int_M \Omega \wedge \bar{\Omega}) \]

and the derivatives are taken with respect to the different moduli

- complex structure (shape of Calabi-Yau) \( z_i \)
- (axion-)dilaton (string coupling) \( \tau = C_0 + ie^{-\phi} \)
- Kähler structure (size of Calabi-Yau) \( \rho_k = b_k + i\sigma_k \)

where \( \sigma_k = \partial_{t_k} \nu \quad \nu = \frac{1}{3!} k_{i,j,k} t^i t^j t^k = \int_M J^3 \)
Flux Compactifications (III)

With a generic choice of fluxes, $F$ and $H$, the complex structure $z_i$ and dilaton $\tau$ are frozen

SUSY condition: \[ D_{z_i} W = D_\tau W = 0 \]

Thus, with \[ W = W_0 \Rightarrow D_{\rho_i} W = W_0 \partial_{\rho_i} K \]

\[ V = e^K (G^{ij} \partial_{\rho_i} K \partial_{\bar{\rho}_j} K - 3)|W_0|^2 = 0 \]

due to cubic dependence on $t_i$ in CY volume in $K$

This is a no-scale model and the Kähler moduli, $\rho_i$, are unconstrained

\[ V \]

\[ V=0 \]

\[ \rho_i \]
A non-trivial potential for the Kähler moduli can be generated in two ways:

- Non-perturbative corrections to $W$
- Perturbative (and non-perturbative corrections) to $K$
Non-perturbative Corrections to $W$ (I)

These effects come in two form:

- **D3-brane instantons (N=1)**
- **Gaugino condensation (N=c_2)**

\[
W = W_0 + \sum_i A_i e^{i a_i \rho_i}, \quad a_i = \frac{2\pi}{N_i}
\]

In the above scenario every divisor $D_i$ has to contribute to $W$, ie the arithmetic genus $\chi(D_i) = 1$

Explicit models shown to exist, though the computation of the complex structure dependence of $A_i(z_j)$ still an issue.

Explicit dependence of $W_0$ on fluxes important
Non-perturbative Corrections to $W$ (II)

One then can solve the F-term equations for the Kähler moduli, which for $h_{11}=1$, gives

$$D_{\rho}W = 0 \quad \Rightarrow \quad t = \frac{\frac{4\pi}{N} V}{(-\frac{W_0}{A})e^{\frac{2\pi}{N}\sigma} - 1}$$

Here we assume $A>0, W_0<0$ and we have already stabilized axion $\text{Re}(\rho)$.

This naturally breaks the no-scale structure of the potential.
Non-perturbative Corrections to $W$ (III)

The conditions on how many divisors (and how they contribute to $W$) can be relaxed

$$W = W_0 + A e^\sum_k i a_k \rho_k , \quad a_k = \frac{2\pi n_k}{N}$$

where the sum is over all $h_{11}$ divisors in $M$.

In this case, a single *ample* divisor $D$ contributes to $W$, where

$$D = \sum_{i=1}^{h_{11}} n_i D_i \iff n_i > 0, \quad i = 1, \ldots, h_{11}$$

As for KKLT one still has to demand that $\chi(D) = 1$
Non-perturbative Corrections to $W$ (IV)

One then can solve the F-term equations for the Kähler moduli, just as in KKLT,

$$D_{\rho_j} W = 0 \quad \Rightarrow \quad t_i = n_i \frac{4\pi}{N} \frac{V}{(-W_0/A) e^{2\pi N \vec{n} \cdot \vec{\sigma}} - 1}, \quad \vec{n} = \{n_i\}$$

Here we assume $A>0, W_0<0$ and we have already stabilized one particular linear combination of the $\text{Re}(\rho_i)$, while the remaining $h_{11}-1$ axions remain massless.

Just as in KKLT, this breaks the no-scale structure
Perturbative Corrections to K (I)

The N=1 Kähler potential receives the first non-zero corrections at $O(\alpha'^3)$ at string tree-level due to the $R^4$ term in the action

$$K_J = -2 \log(V + \frac{\xi}{2} (\frac{\tau - \bar{\tau}}{2i})^{3/2})$$

where

$$\xi = -\frac{\zeta(3)\chi(M)}{2(2\pi)^3}, \quad \xi > 0 \iff \chi(M) < 0$$

Higher order corrections are suppressed by powers and exponentials of the volume of 4-cycles

The $\alpha'^3$ corrections break the no-scale structure just like the non-perturbative corrections to $W$
Perturbative Corrections to K (II)

Due to the correction of the Kähler potential, the metric is no longer (block) diagonal.

One therefore, strictly speaking, has to reanalyze the previous (separate) stabilization of the complex structure moduli, $z_i$, and dilaton, $\tau$.

In particular, it may not be consistent to assume that the F-term constraint vanishes for $\tau$, independently, in the non-susy vacua when (non)-perturbative corrections to $(W) K$ are included.

This leads to additional uplift of the potential, since the metric is positive definite

$$V_{\tau} = e^K (G^{\tau \bar{j}} D_{\tau} W D_{\bar{j}} \bar{W} + G^{i \bar{\tau}} D_i W D_{\bar{\tau}} \bar{W})$$
Perturbative Corrections to K (III)

If \( W = W_0 \)

\[
V = 3\xi \frac{\left( \xi^2 + 7\xi \nu + \nu^2 \right)}{(\nu - \xi)(2\nu + \xi)^2}
\]

where the singular behaviour is due to the inverse Kähler metric \( G^{ij} \)
(Non-)Perturbative Corrections to \((W), K\)

If we include both types of corrections

\[
W = W_0 + \sum_i A_i e^{ia_i \rho_i}, \quad a_i = \frac{2\pi}{N_i}
\]

\[
K_j = -2 \log(V + \frac{\xi}{2} (\frac{\tau - \bar{\tau}}{2i})^{3/2})
\]

the behaviour of the scalar potential at large volume is qualitatively changed

The potential approaches zero from above since perturbative correction to \(K\) dominates in the limit of large volume
The same will be true in the case of a single ample divisor contributing to $W$

$$W = W_0 + A e^{\sum_k i a_k \rho_k}, \quad a_k = \frac{2\pi n_k}{N}$$

$$K_J = -2 \log(V + \frac{\xi}{2} (\frac{\tau - \bar{\tau}}{2i})^{3/2})$$

since once again the behaviour of the scalar potential at large volume is qualitatively changed where the exponential correction to $W$ can be neglected.
Supersymmetry Breaking (I)

Let us first review the argument for how supersymmetry can be broken when $\alpha'$ corrections are included.

We assume that the F-terms vanish for the complex structure moduli, $z_i$, and the dilaton, $\tau$.

Q: When do the F-terms for the Kähler moduli vanish?

\[
D_{\rho_i} W = 0 \quad \Leftrightarrow \quad W_0 = - \left[ \sum_{j=1}^{h_{11}} A_j e^{iaj \rho_j} + 2a_k A_k e^{ia_k \rho_k} \left( \nu + \frac{\xi}{2} \right) \right]
\]

It then follows that, assuming that the volume is large, such that our approximation is valid, a susy minimum will only exist for

\[
|W_0| \leq |W_{max}|
\]
Supersymmetry Breaking (II)

Alternatively, as we increase $|W_0|$, the location of the susy minimum will occur at smaller values of the volume, such that at some point, we will not be able to trust our approximation.

However, as $|W_0|$ is increased further new, non-susy minima will occur at increasingly larger values of the volume.
Supersymmetry Breaking (III)

There are different types of vacua depending on the sign of the scalar potential at the minimum: AdS, Minkowski or dS

As $|W_0|$ is further increased there will be a point where there is no minimum (the dotted curve). At this point, due to the relatively larger perturbative correction to $K_j$ versus the non-perturbative correction of $W$
Supersymmetry Breaking (IV)

How is this changed in the case of a single ample divisor contributing to $W$?

The argument is essentially identical, since

$$D_{\rho_k} W = 0 \iff W_0 = - \left[ A e^{\sum i a_j \rho_j} + 2 a_k A e^{\sum j i a_j \rho_j} \frac{(V + \xi)}{t_k} \right]$$

and there is still an upper bound on $|W_0|$.

Alternatively, following the argument by Bobkov et al, we can change basis to $\rho_D$, $\rho_D = \sum n_i \rho_i$, and a remaining set of linearly independent set of Kähler moduli $\hat{\rho}_i$. In the new basis, independent of the $\alpha'$ correction, the F-terms for the $\hat{\rho}_i$ are always satisfied

$$D_{\hat{\rho}_i} W = 0 \iff W \partial_{\hat{\rho}_i} K = 0$$
Supersymmetry Breaking (V)

Thus, the problem is then reduced to the remaining $\rho_D$ for which we then have

$$D_{\rho_D} W = 0 \iff W_0 = - \left[ A e^{i \frac{2\pi}{N} \rho_D} + 2 \frac{2\pi}{N} A e^{i \frac{2\pi}{N} \rho_D} \left( \mathcal{V} + \frac{\xi}{2} \right) \right]$$

and the argument is identical to that presented before.
Implications

- Allow for uplifting of the potential to be used in cosmological applications such as inflation and the cosmological constant problem.
- No exponentially large volume (unless we relax the ample condition of the divisor).
- Only one axion stabilized, $b_D$, while the remaining $h_{11}$-laxions, $\hat{b}_i$, are massless (they will eventually become massive due to subleading corrections to $W$).
- The $\hat{b}_i$ could then be potential solutions to the strong CP problem.
Discussion and Outlook

- Stringy corrections are important even in the large volume limit
- New mechanism for constructing non-susy vacua with many (nearly) massless axions
- Combine the moduli stabilization with lessons learned from inflationary models and local string constructions
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