Quantum Wires and Simplexes in an Integrable System

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The integrability of delta interaction allows these gates to be chained in a space-time triangle, a simplex.

The unitary connection is re-interpreted (by crossing symmetry) into a gate that connects the sheets of a multiply connected Riemann surface.

This reformulation allows accessible computation in the limit of a large number of degrees of freedom, i.e. a reduction in complexity from \( N! \) to \( N \).
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Delta interaction is an integrable quantum system composed of equivalent point particles in 1(space)+1(time) dimensions.
DYNAMICS

1. Particles travel freely (without acceleration) between elastic collisions that occur when particle coordinates are equal.

2. The particles are distinguishable and their identity is preserved in a collision.

3. A single particle “travels” on a straight world line or “quantum wire”, a line of constant slope in space-time.

4. Two particles travel on two straight world lines that intersect at a single point in space-time, a vertex of the quantum wires.

5. Three particles travel on three straight world lines whose intersections lie at the vertices of a triangle in space-time, a simplex.
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TWO PARTICLE PROBLEM 1

Figure: Two particle diagram
TWO PARTICLE PROBLEM 2

Figure: A delta interaction encounter in space-time

\[ \Psi(x, y) = \begin{cases} f_a e^{ik_ax} e^{ik_by} - f_b e^{ik_bx} e^{ik_ay} & x < y \\ f'_a e^{ik_ax} e^{ik_by} - f'_b e^{ik_bx} e^{ik_ay} & x > y \end{cases} \]
The events “transmission” and “reflection” are complementary

\[ TT^* + RR^* = 1. \]

The events “transmission” and “reflection” are mutually exclusive

\[ TR^* + RT^* = 0. \]

This “transfer matrix” is unitary and symmetric (because the particles are equivalent)
THE THREE PARTICLE PROBLEM 1

Figure: Three particle simplexes

Amplitude for simplex on left = Amplitude for simplex on right

\[ T_{ab} R_{bc} R_{ac} + R_{ab} R_{bc} T_{ac} = R_{ab} T_{bc} R_{ac} \]
THE THREE PARTICLE PROBLEM 2

\[ \frac{T_{ab}}{R_{ab}} + \frac{T_{bc}}{R_{bc}} = \frac{T_{ac}}{R_{ac}}. \]

Satisfied if

\[ \frac{T_{ab}}{R_{ab}} = \frac{i(k_a - k_b)}{g} = i\rho_{ab} \]

\[ T = \frac{i\rho}{i\rho - 1} R = \frac{1}{i\rho = 1} \]
The two particle equations are two relations among four amplitudes:

\[
\begin{pmatrix} f'_a \\ -f'_b \end{pmatrix} = \begin{pmatrix} T_{ab} & -R_{ab} \\ -R_{ab} & T_{ab} \end{pmatrix} \begin{pmatrix} f_a \\ -f'_b \end{pmatrix}
\]

The relations contain the same information if this matrix equation is multiplied by any $2 \times 2$ matrix, or if the matrix is conjugated with any unitary transformation.
The same information is also contained if we rewrite the matrix using crossing symmetry

\[
\begin{pmatrix}
    f'_a \\
    f_a
\end{pmatrix} = \begin{pmatrix}
    1 - \frac{i(k_b-k_a)}{g} & \frac{i(k_b-k_a)}{g} \\
    -\frac{i(k_b-k_a)}{g} & 1 + \frac{i(k_b-k_a)}{g}
\end{pmatrix}
\begin{pmatrix}
    f'_b \\
    f_b
\end{pmatrix}.
\]

The \(2 \times 2\) “transfer matrix” factors into a pair of commuting matrices, leading to a separation of \(a, b\).

\[
\tau(k_a - c_{xy}) \begin{pmatrix}
    f'(k_a) \\
    f(k_a)
\end{pmatrix} = \tau(k_b - c_{xy}) \begin{pmatrix}
    f'(k_b) \\
    f(k_b)
\end{pmatrix} = \begin{pmatrix}
    s_{xy} \\
    1
\end{pmatrix}
\]

and \(c_{xy}, s_{xy}\) are separation constants associated with coordinates \(x, y\).
\[ \tau(k - c) = \left( \begin{array}{cc} 1 - \frac{i(k-c)}{g} & \frac{i(k-c)}{g} \\ - \frac{i(k-c)}{g} & 1 + \frac{i(k-c)}{g} \end{array} \right). \]

Thus,

\[ \frac{f'(k)}{f(k)} = \frac{s + \frac{i(k-c)}{g}(s - 1)}{1 + \frac{i(k-c)}{g}(s - 1)} = \nu((k)). \]

The separation constant \( s^2 = 1 \), because \( \nu \) is a phase. If \( s=1 \), 
\( \nu=1 \) if \( s = -1 \)

\[ \frac{f'(k)}{f(k)} = \frac{2i(k-c)}{g} + 1 = \nu((k)). \]
CONCLUSION

1. No matter how many degrees of freedom the problem is reducible to one particle simplexes.

2. This is a reduction from $N!$ regions of a simply connected region to $N$ sheets of a multiply connected Riemann surface.

Figure: One particle simplex