S-Parameter in the Holographic Walking/Conformal Technicolor

Kazumoto Haba
Nagoya University
19 Dec. 2008 @ Miami 2008

In collaboration with S. Matsuzaki and K. Yamawaki
The Standard model

- The origin of EWSB is one of the main topic which would be uncovered at LHC experiment.

- The solution of SM $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$

- We need to search for the candidates of beyond the Standard model.

One of the candidate is: dynamical symmetry breaking scenario.
QCD-like Technicolor

Weinberg (1976), Susskind (1979)

- EWSB is triggered by Techniquark condensation due to Technicolor interaction at the scale $F_\pi \sim 250$ GeV

- QCD-like TC has some phenomenological problem.

Ex) QCD-like TC gives rise to large contributions to $S$ parameter.

$$S_{EW}^{exp} \lesssim 0.1 \quad \longleftrightarrow \quad S_{TC} \sim 0.3 \times \frac{N_f}{2} \times \frac{N_c}{3}$$

($S_{QCD}^{exp} \sim 0.3$)

S-Parameter

Peskin and Takeuchi (1990), Holdom and Terning (1990), Golden and Randall (1991)

- $S$ is expressed by the vector and axial-vector current correlators $\Pi_V$ and $\Pi_A$:

$$\delta = \frac{S}{N_f/2} = 4\pi \frac{d}{dq^2} \left[ \Pi_V(-q^2) - \Pi_A(-q^2) \right]_{q^2=0}.$$  

- Current correlator

$$\delta^{ab} \left( ig^{\mu\nu} \Pi_J J_{Z'}(p^2) + p^\mu p^\nu \text{term} \right) = \int d^4x e^{-ipx} \langle J_{Z}^{\alpha\mu}(x) J_{Z'}^{\beta\nu}(0) \rangle$$

$Z = 3, Y, V, A$
Walking/Conformal Technicolor

✓ there is an interesting possibility that $S$ decreases in the case of W/C TC!

Appelquist, and Hsu (1992), Sundrum and Hsu (1993),

• An almost non-running (walking) gauge coupling near the conformal fixed point

• Large anomalous dimension of composite operator $\bar{T}T$ especially $\gamma_{\bar{T}T} \simeq 1$

$$\gamma_m \equiv \mu \frac{\partial}{\partial \mu} \ln Z_m, \quad (\bar{T}T)_\Lambda = Z m^{-1} (\bar{T}T)_{\mu}$$
$$\simeq \left( \frac{\Lambda}{\mu} \right)^{\gamma_m} (\bar{T}T)_{\mu}$$

• A typical example of W/C TC is based on $\alpha_*$ the Banks-Zaks infrared fixed point in the large-flavor SU(N) gauge theory, i.e. large-$N_f$ QCD
However, W/C TC has “calculability problem” because perturbative calculations are not reliable.
However, W/C TC has “calculability problem” because perturbative calculations are not reliable.

**Holographic correspondence**  

- **4D strongly coupled theory (4DSCT)** in large $N_c$ limit  
  ![Diagram]

- **5D weakly coupled theory (5DWCT)** at tree level  
  ![Diagram]

- By using Holographic correspondence, Green functions in **4DSCT** can be evaluated from **5DWCT**.

- Bottom-up type “**Holographic QCD**” has succeeded in reproducing several observables of QCD within 30% error.
  
  ![Diagram]
  
  Erlich, Katz, Son and Stephanov (2005), Da Rold and Pomarol, (2005)

- **Holographic Walking/Conformal Technicolor**.
  
  $\gamma^{TT} = 1$ case: Hong and Yee (2007), PiaiL, (2007)
  
outline

• Introduction
• Large $N_f$ QCD with the Banks-Zaks FP
• Holographic Walking/Conformal Technicolor
• Renormalization point of chiral condensation
• Holography vs. W/C TC
• Summary
Two-loop running coupling constant in the large $N_f$ QCD

- Beta function

$$\mu \frac{d}{d\mu} \alpha(\mu) = \beta(\alpha) = -b \alpha^2(\mu) - c \alpha^3(\mu)$$

<table>
<thead>
<tr>
<th>$(N_c = 3)$</th>
<th>$N_f &lt; 8$</th>
<th>$8 &lt; N_f &lt; 16.5$</th>
<th>$16.5 &lt; N_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b = \frac{1}{6\pi} (33 - 2N_f)$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>$c = \frac{1}{12\pi^2} (153 - 19N_f)$</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

- In a range of $8 < N_f < 16.5$ (*the conformal window*)
  - Banks-Zaks IR fixed point $\alpha_*$ (BZFP) appears
- $\alpha \simeq \alpha_*$ the gauge coupling dose not run, but *walks*.
$8 < N_f < 16.5 \quad (\alpha_* = -b/c)$

• $\alpha_*$ becomes smaller as larger $N_f$ in the conformal region.
  ➞ Chiral/Conformal symmetry restores when $N_f \nearrow N_{f\,cr}$ (when $\alpha_* \searrow \alpha_{cr}$).

• At when $\alpha_* = \alpha_{cr}$, $\gamma_{\bar{\ell}T} = 1$.
• $\gamma_{\bar{\ell}T}$ decreases 0 to 1 as $\alpha_* \searrow \alpha_{cr}$
• When $\gamma_{\bar{\ell}T} = 1$, chiral sym. have never been broken.
outline

• Introduction
• Large $N_f$ QCD with the Banks-Zaks FP
• Holographic Walking/Conformal Technicolor
• Renormalization point of chiral condensation
• Holography vs. W/C TC
• Summary
Holographic calculation method

- **4D strongly coupled theory (4DSCT)** in large Nc limit
  - = QCD, TC

- **5D weakly coupled theory (5DWCT)** at tree level

- All 5D bulk fields $\chi(x, z)$ couple to 4D operator $\mathcal{O}(x)$ at UV boundary:
  - Operator: $\mathcal{O}(x)$
  - Field: $\chi(x, z)$

- 5D effective action corresponds to generating functional of $\mathcal{O}(x)$

$$
\langle e^{\int d^4 x \chi_0(x) \mathcal{O}(x)} \rangle = e^{S_{\text{eff}}[\chi^{cl}(x, z=0)]}
$$

Then UV boundary value of $\chi^{cl}(x, z)$ corresponds to source of $\mathcal{O}(x)$.

$$
\chi_0(x) \sim \chi^{cl}(x, z)|_{z=0}
$$


UV brane

$z = 0$

$A_\mu, V_\mu, \phi$
Set up of a Holographic W/C Technicolor model

• This model is based on a Holographic QCD model.

**4D Strongly Coupled Theory**

- $\text{SU}(N_f)_L \times \text{SU}(N_f)_R$ *global* symmetry

- 4D operators: L-, R-current $j^L_\mu$, $j^R_\mu$; chiral operator $\bar{q}q$

**5D Weakly Coupled Theory**

- $\text{SU}(N_f)_L \times \text{SU}(N_f)_R$ *gauge* symmetry

- Bulk fields coupled to 4D operators, respectively: L-, R-gauge field $L_\mu$, $R_\mu$; scalar field $\phi$

Erlich, Katz, Son and Stephanov (2005), Da Rold and Pomarol, (2005)
Set up of an Holographic W/C walking Technicolor

**5D Weakly Coupled Theory**

- chooses AdS5 metric:
  \[ ds^2 = a^2(z) \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right), \quad a(z) = \frac{L}{z} \]

- compactifies 5th direction "z" from \( \epsilon \) to \( z_m \):
  \[ \epsilon \leq z \leq z_m \]

We need \( z_m \) to introduce a mass gap in the theory.

- is described by the action:
  \[ S_5 = \int d^5 x \sqrt{g} \frac{1}{g_5^2} \text{Tr} \left[ -\frac{1}{4} F_{L,R}^2 + \frac{1}{2} |D_M \Phi|^2 - \frac{1}{2} m_5^2 |\Phi|^2 \right] \]

  \[ D_M \Phi = \partial_M \Phi + i L_M \Phi - i R_M \Phi, \quad \text{Tr}[T^a T^b] = \delta_{ab} \]

UV brane \( z = \epsilon \to 0 \) \( L_\mu, R_\mu, \phi \) IR brane \( z = z_m \) Z
Anomalous dimension in Holographic W/C TC

- Bulk scalar mass $m_5$ relates $\gamma_m$:

  $$m_5^2 = \frac{\Delta(\Delta - 4)}{L^2},$$

Conformal dimension of $\langle \bar{T}T \rangle$:

  $\Delta = 3 - \gamma_m$

- In this model, d.o.f $\gamma_m$ is replaced with mass of a 5-dimensional bulk scalar field as a free parameter.
The generating functional of $\mathcal{O}(x)$ relates the effective action expressed as a functional of the UV boundary value $\chi^{cl}(x, \epsilon)$. 

$$S_{\text{eff}}[V^{cl}, A^{cl}, S^{cl}_0] \sim \int_x \text{Tr} \left[ v^\mu \Pi_V v_\mu + a^\mu \Pi_A a_\mu + M_T \bar{q}q \right]$$

$$V_M, A_M = \frac{1}{\sqrt{2}} (L_M \pm R_M), \quad S_0 = \langle \Phi \rangle$$

Chooses boundary condition:

\begin{align*}
\text{UVBC} & \\
V_\mu, A_\mu |_\epsilon & \equiv v_\mu, a_\mu \\
\left( \frac{L}{\epsilon} \right)^{1+\gamma_m} S_0 |_\epsilon & \equiv M_T \\
M_T: \text{Current (techni)quark mass} \\
\text{IRBC} & \\
\partial_z (V_\mu, A_\mu) |_{z_m} & = 0 \\
L S_0 |_{z_m} & \equiv \xi
\end{align*}

$\xi$ plays role of Chiral condensation

$$\langle \bar{T}T \rangle = \frac{L}{g_5^2} \frac{3 - \gamma_m}{2z_m^3} \left( \frac{z_m}{L} \right)^{\gamma_{TT}} \xi$$
Parameters in the Holographic W/C Technicolor

- This model has 6 parameters

<table>
<thead>
<tr>
<th>Anomalous dimension</th>
<th>Ultraviolet cutoff</th>
<th>Infrared cutoff</th>
<th>Current mass</th>
<th>IR boundary Value of $S_0$</th>
<th>5D gauge coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma T^T$,</td>
<td>$\epsilon$,</td>
<td>$z m$,</td>
<td>$M_T$,</td>
<td>$\xi$,</td>
<td>$L / g_5^2$</td>
</tr>
</tbody>
</table>
Parameters in the Holographic W/C Technicolor

- This model has 6 parameters,
  \[ \gamma \bar{T} T, \quad \epsilon, \quad z_m, \quad M_T, \quad \xi, \quad L/g_s^2. \]

- 4 undetermined parameters
  \[ N_{TC}, \quad \gamma \bar{T} T, \quad 1/M_\rho, \quad \xi. \]

- Since S-Parameter is dimensionless, S should depend on 3 dimensionless parameters, \( N_{TC}, \gamma \bar{T} T \) and \( \xi \):
  \[ \mathcal{S}_{\text{horo}} = \mathcal{S}( N_{TC}, \gamma \bar{T} T, \xi ) \]
\( \xi \) \textbf{with respect to} \( (F_\pi/M_\rho) \)

- \( (F_\pi/M_\rho) \) depends on \( N_{TC}, \gamma_{\bar{T}T} \) and \( \xi \):

\[
\xi \gg 1 : \quad \xi^2 \approx \left( \frac{C_1/\xi(\gamma_{\bar{T}T})}{N_{TC}} \cdot \frac{F_\pi^2}{M_\rho^2} \right)^{3-\gamma_{\bar{T}T}} \\
\xi \ll 1 : \quad \xi^2 \approx \frac{C_\xi(\gamma_{\bar{T}T})}{N_{TC}} \cdot \frac{F_\pi^2}{M_\rho^2}
\]

**FIG. 2:** Plot of \( \xi^2 \)-dependence of \( \frac{f_\pi}{\sqrt{N_{TC}}} z_m \) with \( \gamma_m \approx 1 \).

- \( (F_\pi/M_\rho) \) \textbf{becomes larger as} \( \xi \) \textbf{larger}
- \textbf{v.v.} \( \xi \) can be expressed by \( N_{TC}, \gamma_{\bar{T}T} \) together with \( (F_\pi/M_\rho) \):

Parameter \( \xi \) can be almost identified as \( (F_\pi/M_\rho) \)
The change of $\gamma TT$ does not give a dominant contribution to $\hat{S}$ (at least in the small region of $\xi \leftrightarrow (F_\pi/M_\rho)^2$).

In other words, only the ratio $(F_\pi/M_\rho)$ does dominate $\hat{S}$.
(\(f_\pi/M_\rho\)) - Contribution to \(\hat{S}\)

Contours corresponding to \(\hat{S}\) drawn on \((\gamma_{TT},\xi)\)-plane with \(N_{TC} = 3\) fixed.

Plot of \((F_\pi/M)\) dependence of \(S\) with \(\gamma_{TT} \sim 1\) and \(N_{tc} = 3\) fixed.

- \(\hat{S}\) is given as a monotonic & positive function of \(\xi\), or equivalently, of \((F_\pi/M_\rho)\) for each value of \(\gamma_{TT}\).

- \(\hat{S} \to 0\) is achieved by taking \(\xi \to 0\) for an arbitrary \(\gamma_{TT}\).

\((F_\pi/M_\rho) \to 0\)
outline

• Introduction
• Large $N_f$ QCD with the Banks-Zaks FP
• Holographic Walking/Conformal Technicolor
• Renormalization point of chiral condensation
• Holography vs. W/C TC
• Summary
Renormalization point of chiral condensation

• In holographic framework, the current quark mass is related to UV boundary value of bulk scalar field $S_0$.

$$M_T = \left( \frac{1/L}{1/\epsilon} \right)^{-\gamma \bar{T}T} \cdot \left( \frac{L}{\epsilon} v(\epsilon) \right)$$

UVBC:

• This relationship is the same to “renormalization” of the current quark mass

$$[M_T]_{1/L} = \left( \frac{1/L}{1/\epsilon} \right)^{-\gamma \bar{T}T} \cdot [M_T]_{1/\epsilon}$$

• We may regard this $M_T$ as the one “renormalized” at the scale $(1/L)$.

• $\langle \bar{T}T \rangle$ is “renormalized” at the scale $(1/L)$ as well.

$$\langle \bar{T}T \rangle_{1/L} = \frac{L}{g_5^2} \frac{3 - \gamma_m}{2z_m^3} \left( \frac{1/M_\rho}{L} \right)^{\gamma \bar{T}T} \xi$$
Renormalization point of chiral condensation- II

- The scaling-law of $\langle \bar{T}T \rangle_{1/L}$ is given by

$$\langle \bar{T}T \rangle_{1/L} \simeq \left(\frac{1/L}{m}\right)^{\gamma_{TT}} \langle \bar{T}T \rangle_m \simeq N_{TC} \cdot \left(\frac{1/L}{m}\right)^{\gamma_{TT}} m^3$$

- $m$ denotes the chiral symmetry breaking scale:

$$\langle \bar{T}T \rangle_m \simeq N_{TC} \cdot m^3$$

- Typically $m$ is a dynamical mass of techniquark.

- $\xi$ DOES NOT depend on the renormalization scale $(1/L)$

$$\xi = \frac{24\pi^2}{N_{TC}} \frac{1/M^3}{3-\gamma_{TT}} \left(\frac{M}{1/L}\right)^{\gamma_{TT}} \langle \bar{T}T \rangle_{1/L} \simeq \frac{24\pi^2}{3-\gamma_{TT}} \left(\frac{m}{M}\right)^{3-\gamma_{TT}}$$

- $\hat{S}(N_{TC}, \gamma_{TT}, (F_\pi/M)^2 \Leftrightarrow \xi)$ DOES NOT depend, as it should.
The scaling-law of $F_\pi$

- A scaling relation between $F_\pi$ and $m$, as chiral restoration for $\xi \gg 1$ and $\xi \ll 1$:

$$(F_\pi/M_\rho) \gg 1 \ : \ F_\pi^2 \sim m^2,$$

$$(F_\pi/M_\rho) \ll 1 \ : \ F_\pi^2 \sim m^2 \cdot \left(\frac{m}{M_\rho}\right)^{4-2\gamma_{TT}},$$

$\xi$ with respect to $\left(\frac{F_\pi}{M_\rho}\right)$ in the limit of $\xi$:

$\xi \gg 1 \ : \ \xi^2 \sim \left(\frac{F_\pi^2}{M_\rho^2}\right)^{3-\gamma_{TT}}$

$\xi \ll 1 \ : \ \xi^2 \sim \frac{F_\pi^2}{M_\rho^2}$

- Note that

- QCD case is the case of $\xi \gg 1$ with $\gamma_{TT} \simeq 0$ and $N_c = 3$ fixed. 

  Da Rold and Pomarol, (2005)

  The well-known scaling law can be reproduced:

  $$F_\pi^2 \sim m^2$$

- The scaling law for $(F_\pi/M_\rho) \ll 1$ is a novel scaling-law!!
outline

• Introduction
• Large $N_f$ QCD with the Banks-Zaks FP
• Holographic Walking/Conformal Technicolor
• Renormalization point of chiral condensation
• Holography vs. W/C TC
• Summary
One might think...

• The value of the ratio \( \frac{F_\pi}{M_\rho} \) is fixed to be \( \mathcal{O}(1) \), as in the case of QCD, and hence that \( \tilde{S} = \text{constant} \neq 0 \)

However,

• It might be non-trivial that \( F_\pi \) and \( M_\rho \) simultaneously go to zero as chiral/conformal restoration.

• Even in the case of Large \( N_f \) QCD, several scaling relations have been clarified...

We provide an interpretation for the result of the holographic calculation in term of

scaling behaviors as the chiral conformal phase transition.
Searching For Explicit Dynamics

• We will discuss how the conformal phase transitions of 3 types can be realized in an explicit W/C TC dynamics.

When $\alpha_* \rightarrow \alpha_{cr}$, chiral/conformal sym. restores

\[ \left( \frac{f_\pi}{M_\rho} \right) \]

Chiral Restoration

Sym.

\[ M_\rho \downarrow 0 \text{ faster than } F_\pi \downarrow 0 \text{ as } \alpha_* \rightarrow \alpha_{cr}. \]
\[ \bar{S} \text{ grows to diverge when } F_\pi = 0. \]

\textbf{large } N_f \text{ QCD based on Hidden Local Sym.}

\[ M_\rho \downarrow 0 \text{ as fast as } F_\pi \downarrow 0 \text{ as } \alpha_* \rightarrow \alpha_{cr}. \]
\[ \bar{S} = \text{constant, even when } F_\pi = 0. \]

\textbf{large } N_f \text{ QCD based on SD Eq. & BS Eq.}

\[ M_\rho \downarrow 0 \text{ slower than } F_\pi \downarrow 0 \text{ as } \alpha_* \rightarrow \alpha_{cr}. \]
\[ \bar{S} \text{ decreases, resulting in } \bar{S} = 0 \text{ when } F_\pi = 0. \]

Unknown...
An Example of iii) : unknown

• This case may rearized if the part of Mρ consists of the gluon condensation.

\[
\frac{M_\rho}{F_\pi} \rightarrow \left( \frac{M_\rho^{(\bar{T}T)\text{part}} \rightarrow 0}{F_\pi^{(\bar{T}T)\text{part}} \rightarrow 0} \right) + M_\rho^{\text{gluball part}} \rightarrow \infty
\]

• in this case, S becomes exactly “0” when chiral symmetry restores!!

• Is this trivial because S is LR current mixing?

>>> No !!

In a perturbative calculation of S, S becomes constant even when chiral restoration occurs.

\[
S_{QCD}^{\text{pert}} = \frac{N_c N_f}{6\pi} \quad \text{even when chiral restoration occurs.}
\]

• Holographic calculation has provided us with another scaling-law of \( F_\pi \)

\[
F_\pi^2 \sim m^2 \cdot \left( \frac{m}{M_\rho} \right)^{4-2\gamma_{\bar{T}T}}
\]

- \( \xi \gg 1 : F_\pi^2 \sim m^2 \)
- \( \xi \ll 1 : F_\pi^2 \sim m^2 \cdot \left( \frac{m}{M_\rho} \right)^{4-2\gamma_{\bar{T}T}} \)
Summary

• Based on holographic correspondence, we have studied the $\hat{S}$ in models of W/C TC in a wide range of $\gamma_{TT}$: $0 \leq \gamma_{TT} < 1$.

• $\hat{S}$ is given as a function of $(F_\pi/M_\rho)$ and that the shift of the value of $\gamma_{TT}$ from 0 to 1 does not give a dominant contribution to $\hat{S}$.

• We classified holographic W/C TC models into three cases, in terms of the scaling behaviors of $F_\pi$ and $M_\rho$.

• Especially in the case that $M_\rho \ll 0$ slower than $F_\pi \ll 0$, it was shown that $\hat{S}$ goes to exactly zero at the edge of the conformal window for an arbitrary $\gamma_{TT}$.

• In this case, by identifying $1/L$ as the renormalization point of $\langle \bar{T}T \rangle_{1/L}$ we found a novel scaling property of $F_\pi$ with respect to $m$

\[ f_\pi^2 \sim m^2 \cdot \left( \frac{m}{M_\rho} \right)^{4-2\gamma_{TT}} \]
S in the limit of  \( \xi \gg 1 \) and  \( \xi \ll 1 \)

- S-Parameter depends on 3 parameters:

\[
\hat{S}_{\text{holo}} = \hat{S}(NT_{TC}, \gamma_{\bar{T}T}, (F_\pi/M_\rho)^2 \leftrightarrow \xi)
\]

- We can analytically estimate S in powers of  \( \xi \) or  \( \xi \ll 1/\xi \)

  \( \xi \gg 1 \) : 
  \[
  \hat{S} \simeq \frac{NT_{TC}}{3} \ln(F_\pi/M_\rho)^2,
  \]

  \( \xi \ll 1 \) : 
  \[
  \hat{S} \simeq \frac{C_\xi(\gamma_{\bar{T}T})}{6\pi} \frac{(4 - \gamma_{\bar{T}T})}{(3 - \gamma_{\bar{T}T})^2} \left(\frac{F_\pi}{M_\rho}\right)^2
  \]

It should be noted that

- \( \hat{S} \) is proportional to  \( (F_\pi/M_\rho)^2 \) in the small region of  \( \xi \) for an arbitrary  \( \gamma_{\bar{T}T} \).

- \( \hat{S} \to 0 \) is achieved by taking  \( (F_\pi/M_\rho) \to 0 \) for an arbitrary  \( \gamma_{\bar{T}T} \).
Vector current correlator

• Vector current correlator

\[ \Pi_V(p^2) = -\frac{L}{g_5^2} \frac{i|p| Y_0(i|p|z_m)J_0(i|p|\epsilon) - J_0(i|p|z_m)Y_0(i|p|\epsilon)}{\epsilon Y_0(i|p|z_m) J_1(i|p|\epsilon) - J_0(i|p|z_m)Y_1(i|p|\epsilon)} \]

• Matching to Operator Product Expansion result

\[ \Pi_V(|p|^2 \to \infty) \sim \frac{L|p|^2}{2g_5^2} \log (|p|^2 \epsilon^2) \quad \iff \quad \frac{N_c}{24\pi^2} \log (|p|^2 \epsilon^2) \]

\[ \frac{L}{g_5^2} = \frac{N_{TC}}{12\pi^2} \]

• The lowest pole can be identified as techni- \( \mathcal{Q} \) mass

\[ z_m \approx \frac{2.4}{M_\rho} \quad \iff \quad \Pi_V(|q|^2) = -|q|^2 \sum \frac{F_{Vn}^2}{|q|^2 + M_{Vn}^2} \]
axial current correlator

- The axial current correlator can similarly be calculated. Especially
  - $F_\pi$ is calculated to be

$$F_\pi^2 \equiv -\Pi_A(0),$$

$$= \frac{L}{g_5^2 \epsilon^2} \Delta X(\epsilon) \frac{I_{1-\Delta}(X(z_m)) \cdot K_{1-\Delta}(X(\epsilon)) - K_{1-\Delta}(X(z_m)) \cdot I_{1-\Delta}(X(z))}{I_{1-\Delta}(X(z_m)) \cdot K_{1-\Delta}(X(\epsilon)) + K_{1-\Delta}(X(z_m)) \cdot I_{1-\Delta}(X(\epsilon))}.$$

- S-Parameter is given by

$$S = 4\pi \frac{d}{dp^2} \left[ \Pi_V(-p^2) - \Pi_A(-p^2) \right] \bigg|_{p^2=0}$$

$$\approx \frac{L}{g_5^2} \int_\epsilon^{z_m} \frac{dz'}{z'} \left[ 1 - \left( \frac{z'}{\epsilon} K_{1/\Delta} \left( \frac{\sqrt{2}\zeta}{3z_m^2 z^2} \right) \right)^2 \right]$$

Let me more easily explain these complex expressions.