We discuss the on-shell $N = 1$ supersymmetric coupling of brane chiral multiplets in the context of $N = 2$, $D = 5$ Supergravity compactified on $S_1/Z_2$ orbifolds. Assuming a constant superpotential on the hidden brane, we study the transmission of the supersymmetry breaking to the visible brane. We find that to the lowest order in the five dimensional Newton’s constant $\kappa_5^2$ and gravitino mass $m_{3/2}^2$ the spinor field of the radion multiplet is responsible of inducing positive one – loop squared masses $m_\varphi^2 \sim m_{3/2}^2 / (M_{\text{Planck}}^2 R^2)$ to the scalar fields which are localized on the visible brane with $R$ the length scale of the fifth dimension. Considering a cubic superpotential on the visible brane we also find that non vanishing soft trilinear couplings $A$ are induced given by $A = 3m_\varphi^2 / m_{3/2}^2$. 
1. Introduction

One of the main issues that may be adressed in supersymmetric brane world models is the MEDIATION OF THE SUPERSYMMETRY BREAKING and the DETERMINATION OF THE SOFT- BREAKING TERMS appearing in the 4D low energy theories.

[Mirabelli-Peskin-Bagger-Feruglio-Ghergetta-Riotto-Zwirner…]

These models may be constructed by orbifolding a supersymmetric theory with a compact extra dimension.

The Supersymmetry breaking is triggered on the hidden brane and then through the bulk is communicated to the visible brane inducing for example finite one loop corrections to the scalar mass of the scalar fields that live on the visible brane.

[Ghergetta-Riotto-Rattazzi-Srucca-Strumia]

In our work we study the transmission of the Supersymmetry breaking in $N = 2, D = 5$ Supergravity compactified on $S_1/\mathbb{Z}_2$ orbifold.

The visible brane is at $x^5 = 0$ and the hidden one at $x^5 = \pi R$.

We construct the $N = 1$ supersymmetric coupling of brane chiral multiplet with the bulk fields by working directly in the ON - shell scheme.
2. THE MODEL

- 5D Supergravity multiplet

\[ \{e^{\tilde{m}}_{\mu}, \Psi^i_{\tilde{\mu}}, A_{\tilde{\mu}}\} \]

Funbein \( e^{\tilde{m}}_{\mu} \) two gravitini \( \Psi^i_{\tilde{\mu}} \)

graviphoton \( A_{\tilde{\mu}} \)

\[ \tilde{\mu} = (\mu,5) \text{curved} \]
\[ \tilde{m} = (m,5) \text{flat} \]

\( i = 1,2 \) symplectic \( SU(2)_R \) index
The 5D ON - SHELL Langrangian

\[ \mathcal{L}_0 / e^{(5)} = -\frac{1}{2} R^{(5)} + \frac{i}{2} \overline{\Psi}_\mu \gamma^{\mu\nu\rho} \nabla_{\nu} \Psi^i_{\rho} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]

\[ + \frac{3i}{8\sqrt{6}} \left[ \overline{\Psi}_\mu \gamma^{\mu} \gamma^{\nu} \gamma^{\rho} \gamma^{\tilde{\sigma}} \Psi^i_{\nu} F_{\rho\tilde{\sigma}} + 2 \overline{\Psi}_\mu \Psi^i_{\nu} F^{\mu\nu} \right] \]

\[ + \frac{\epsilon^{\mu\nu\rho\sigma\lambda}}{6\sqrt{6} e^{(5)}} F_{\mu\nu} F_{\rho\sigma} A_{\lambda} + (4 - \text{fermion}) \]

[Cremmer-Niewenhuizen-Gunaydin-Sierra-Townsend-Van Proyen-de Wit-Nikolai…]

**S\textsubscript{1}/Z\textsubscript{2} orbifold**

The 5th dimension is considered to be compact (circle) and the S\textsubscript{1}/Z\textsubscript{2} orbifold is realized by assigning appropriate Z\textsubscript{2} parity to the fields

**Z\textsubscript{2} – even fields** \(e^m_{\mu}, e^5_{\mu}, \Psi^1_{\mu}, \Psi^2_{\mu}, A_5\)

**Z\textsubscript{2} – odd fields** \(e^5_{\mu}, e^m_{\mu}, \Psi^2_{\mu}, \Psi^1_{\mu}, A_{\mu}\)
orbifolding

N=2 SUGRA  \rightarrow  N = 1 SUGRA
D = 4  on the branes

4-D supergravity multiplet

\{ e^m_\mu, \psi_\mu \equiv \psi^1_\mu \} \quad \text{Where } \psi^1_\mu \equiv \Psi^1_{\mu L}

Radion multiplet

\[ T \equiv \left( \frac{1}{\sqrt{2}} \left[ e_5^5 - \frac{i\sqrt{2}}{\sqrt{3}} A_5^\circ \right] \right), \chi^{(T)} \equiv -\psi^2_5 \]

N = 1 SUSY is the one corresponding to the even \( \varepsilon \) parameter.
What we want to do:

- Write the $N = 1$ supersymmetric couplings of the radion field with the chiral multiplet $(\phi, \chi)$ that lives on the visible brane at $x^5 = 0$.

In a previous work we have derived the coupling of the supergravity multiplet with the chiral multiplet, ignoring the radion multiplet.

[Diamandis-Georgalas-Kouroumalou-Lahanas]

- Nöther procedure turns out to be cumbersome - many terms arise.

- **Alternatively**: write the couplings using standard knowledge of $N = 1$, $D = 4$ SUGRA.
- Since the restriction of the radion field on the brane forms a chiral multiplet, we can seek for a Kähler function \( F \) to describe the coupling in the usual manner in \( N = 1, D = 4 \) SUGRA. \[ Zumino, Bagger, Witten \]

- We can split \( F \) as:

\[
F = \mathcal{N}(T, T^*) + K(T, \varphi, T^*, \varphi^*)
\]

Restriction of 5D SUGRA on the brane (survives in the absence of brane multiplets)

Associated to brane multiplets

\[
\equiv \Delta_{(5)} K(\varphi, \varphi^*)
\]

\[
\Delta_{(5)} \equiv e_5^5 \delta(x^5)
\]

-Determination of \( \mathcal{N}(T, T^*) \):

• The \( N = 2, D = 5 \) SUGRA that lives in the bulk, when restricted on the branes doesn't provide a typical \( N = 1, D = 4 \) SUGRA.

Because:

All terms in

~ the 5D – action involve \( e_5^5 \) instead of \( e_5^{(4)} \) and

~ there is NO kinetic terms to \( \psi_5^2 \) and
This is achieved at least at $O(\kappa_5^2)$ by the following transformations on the fields on the original 5D Langrangian:

- $e^m_\mu \rightarrow e^f e^m_\mu$
- $\psi^{1,2}_{\mu} \rightarrow e^{f/2} \left( \psi_{\mu} + \frac{i}{2} \sigma_{\mu} \psi^{2,1}_5 e^{2f} \right)$
- $\psi^{1,2}_5 \rightarrow e^{-f/2} \psi^{1,2}_5$

where $e^{2f} \equiv e^5$

In terms of the transformed fields the couplings of the brane fields with the radion multiplet are those of the $N = 1, D = 4$ Supergravity, determined by the Kähler function:

$$\mathcal{F} = -3 \ln \frac{T + T^*}{\sqrt{2}} + \delta(x^5) \frac{\sqrt{2}}{T + T^*} K(\varphi, \varphi^*)$$

That done the Langrangian including the interaction of the brane fields with the radion multiplets is:
INTERACTION OF BRANE FIELDS WITH RADION MULTIPLET

\[
\mathcal{L}_o = -e^{(4)}(\mathcal{F}_{ij}^\ast \left[ \partial_\mu \phi^i \partial^\mu \phi^{*j} + \frac{i}{2} \left( \chi^i \sigma^\mu D_\mu \bar{\chi}^j + \bar{\chi}^j \sigma^\mu D_\mu \chi^i \right) \right] \\
- \frac{1}{4} e^{(4)}(\mathcal{F}_{ij}^\ast (\mathcal{F}_m \partial_\mu \phi^m - \mathcal{F}_m \partial^\mu \phi^{*m}) - 2(\mathcal{F}_{ij}^\ast \partial_\mu \phi^m - \mathcal{F}_{ij}^\ast \partial^\mu \phi^{*m})\chi^{*i} \sigma^\mu \bar{\chi}^j \\
- \frac{1}{\sqrt{2}} e^{(4)}(\mathcal{F}_{ij}^\ast \partial_\nu \phi^* \chi^i \sigma^\mu \bar{\sigma}^\nu \psi_\mu + h.c.) + \\
+ \left( \frac{e^{(4)}}{4} \right) \left( \mathcal{F}_m \partial_\kappa \phi^m - (\mathcal{F}_m \partial_\kappa \phi^{*m}) \psi_\chi \sigma_\mu \bar{\psi}_\nu \right) \\
+ \frac{e^{(4)}}{4} (\mathcal{F}_{ij}^\ast r_{\kappa \nu} \psi_\chi \sigma_\mu \bar{\psi}_\nu + \psi_\mu \sigma_\nu \bar{\psi}_\mu) \chi^i \sigma^\nu \bar{\chi}^j + \\
+ \frac{e^{(4)}}{16} \chi^i \sigma^\mu \bar{\chi}^j (\mathcal{F}_{ij}^\ast (\mathcal{F}_{kl}^\ast - 2R_{ij}^\ast \kappa) \chi^i \sigma^\mu \bar{\chi}^j \chi^k \sigma^\mu \bar{\chi}^l) \quad i, j = T, \phi_i
\]

One can easily add a non trivial superpotential \( W(\phi) \) giving rise to Yukawa and potential terms

NON TRIVIAL SUPERPOTENTIAL \( W(\phi) \):

\[
\mathcal{L}_{Yukawa} + \mathcal{L}_p = \left. e^{(4)} \Delta_{(5)}^i e^{F_i/2} \right. \left( W^* \psi_\mu \sigma^{\mu
u} \psi_\nu + \frac{i}{\sqrt{2}} D_i \chi^i \sigma^\mu \bar{\psi}_\mu + \frac{1}{2} D_i D_j \chi^i \bar{\chi}^j + h.c. \right) \\
- \left. e^{(4)} \left( \Delta_{(5)} \right)^i e^{F_i} \left( \mathcal{F}_{ij}^\ast D_i \bar{W} \bar{D}_j W^* - 3|W|^2 \right) \right.
\]
\( \mathcal{L}_o \) contains both terms describing the interaction of the radion multiplet with the fields localized on the brane and also terms involving only the radion multiplet fields which live in the bulk.

This is due to the particular form of the Kähler metric arising from the Kähler function \( \mathcal{F} \):

\[
(\mathcal{F}_{TT^*}) = \frac{3}{(T + T^*)^2} + \delta(x^5) \frac{2\sqrt{2}}{(T + T^*)^3} K = \left( e_5^* \right)^2 \left( \frac{3}{2} + \Delta_{(5)} K \right)
\]

\[
(\mathcal{F}_{\phi T^*}) = -\frac{1}{\sqrt{2}} e_5^* \Delta_{(5)} K_{\phi^*},
\]

\[
(\mathcal{F}_{T \phi^*}) = -\frac{1}{\sqrt{2}} e_5^* \Delta_{(5)} K_{\phi^*},
\]

\[
(\mathcal{F}_{\phi \phi^*}) = \Delta_{(5)} K_{\phi^*}
\]
II. Similar expression holds for the hidden brane at $x^5 = \pi R$. We just have to add a hidden part Kähler function

$$\mathcal{F}_H = \delta(x^5 - \pi R) \frac{\sqrt{2}}{T + T^*} K_H(\phi_H, \phi_H^*)$$

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to depending only on the hidden brane fields and the corresponding $W_H$.

III. The extra power of the $\Delta_{(5)}$ prefactor multiplying the potential terms is cancelled in the first term since the inverse of the Kähler metric includes the inverse $(\Delta_{(5)})^{-1}$. However it is not cancelled in the second term proportional to $|W|^2$.

Not new – also occurs and in $N = 2$, $D = 5$ supersymmetries in FLAT space – time where they are singularities arising from the propagation of the bulk fields in order to maintain supersymmetry.

[Mirabelli, Peskin]
IMPORTANT!

The negative term $-3|W|^2$ flips its sign in the scalar potential if we consider the coupling with the radion multiplets for example:

For $W(\varphi) = \frac{\lambda}{6} \varphi^3$ and $K(\varphi, \varphi^*) = \varphi \varphi^*$

then

$$\mathcal{L}_p = -e^{(4)} \Delta_{(5)} e^{\mathcal{F}} \left( \frac{|\lambda|^2}{4} \varphi^2 \varphi^* + \Delta_{(5)} \frac{|\lambda|^2}{12} \varphi^3 \varphi^* \right)$$

$\nabla$ always positive independent of the form of $K, W$
3. TRANSMISSION OF THE SUSY BREAKING

We consider a constant superpotential $c$ on the hidden brane. The corresponding Langrangian is:

$$\mathcal{L}_h = -e^{(4)} \Delta^{(h)}_{(5)} e^{N/2} \left( c^* \psi_\mu \sigma^{\mu \nu} \psi_\nu + \frac{3i}{2} c \psi_5^2 \sigma^\mu \overline{\psi}_\mu + \frac{3}{2} c \psi_5^2 \psi_5^2 + h.c. \right)$$

$$\Delta^{(h)}_{(5)} = \delta(x_5 - \pi R) e_5^5$$

- Mass terms for $\psi_\mu$ and $\psi_5^2$ arise on the hidden brane.

- We choose $K = \varphi \varphi^*$ on the visible brane.

- Gauge choice:
The one in which the bulk kinetic terms of the gravitinos are disentangled from their fifth components.

- Treat mass terms on the hidden brane as interaction terms (sufficient for our purposes $O(m_{3/2}^2) \propto |c|^2$).

Or schematically:
Gravitino and $\psi_5^2$
Mass terms on HIDDEN BRANE

$\Delta_{(5)} K \left( \psi_5^2 \sigma^\mu D_\mu \bar{\psi}_5^2 + \bar{\psi}_5^2 \bar{\sigma}^\mu D_\mu \psi_5^2 \right)$
+ gauge fixing term
+ diagonalization of kinetic terms

Yields $\Delta m_\phi^2$

- Gauge fixing term add to the bulk Langrangian

$$i \frac{\xi}{2} \hat{\Psi}_{\bar{m}} \gamma^\gamma \gamma^\gamma \partial \hat{\Psi}_{\bar{n}}$$
\( \tilde{m} = (m, \hat{5}) \) Flat 5D indices

\( \hat{\Psi}^i_{\tilde{m}} \) i = 1, 2 denote 4-component gravitinos in the original 5-D Langrangian (symplectic Majorana gravitinos of 5D, N = 2 SUGRA).

- **Gauge choice** \( \zeta = -\frac{3}{4} \)

- Additional shift: \( \psi^{1,2}_{\hat{5}} \rightarrow \psi^{1,2}_{\hat{5}} \pm \frac{i}{3} \sigma^m \bar{\psi}^{2,1}_m \)

eliminates \( m, \hat{5} \) mixings

- Define Dirac spinors: \( \Psi = \begin{pmatrix} \psi^2_{\hat{5}} \\ \psi^1_{\hat{5}} \\ \psi^1_{\tilde{m}} \\ \psi^2_{\tilde{m}} \end{pmatrix}, \Psi_m = \begin{pmatrix} \psi^1_m \\ \psi^2_m \end{pmatrix} \)
ORBIFOLDED PROPAGATORS IN MIXED MOMENTUM - CONFIGURATION

[Arkani-Hamed-Nomura-Weiner-Puchwein-Nilles]

\[ G_{mn}(p, y, y') = \left( \frac{1}{2} \gamma_n P \gamma_m + i n_{mn} \gamma^5 \partial y \right) F(p, y, y') \]

\[ G(p, y, y') = \frac{2i}{9} \left( p + i \gamma^5 \partial y \right) F(p, y, y') \]

\[ F(p, y, y') \equiv \frac{1}{2q \sin(q \pi R)} \left\{ \cos \left[ q \left( \pi R - |y - y'| \right) \right] - \gamma^5 \cos \left[ q \pi R - (y + y') \right] \right\} \]

Where \( q = \sqrt{-p^2 + i \varepsilon} \) and \( y, y' \) are along the fifth dimension. So

**VISIBLE BRANE**

\[ -i \varphi^* \left[ \overline{\Psi}_R \partial P_L \Psi + \frac{1}{9} \Psi_m P_R \gamma^m \partial \gamma^n P_L \Psi + i \frac{1}{3} \left( \Psi^T P_L C \gamma^m \partial P_L \Psi - hc \right) \right] \]

**HIDDEN BRANE**

\[ -\frac{1}{8} \left[ \Psi^T P_L C \left( \gamma^{mn} - \frac{1}{3} \gamma^m \gamma^n \right) P_L \Psi + \frac{3}{2} \Psi^T P_L C P_L \Psi - i \frac{1}{2} \overline{\Psi}_R \gamma^m P_L \Psi + hc \right] \]

Where \( P_{L,R} = \frac{1}{2} \left( 1 \pm i \gamma^5 \right) \)
Correction to the scalar masses

Diagrams relevant for the calculation of the induced scalar field masses. The dashed lines denote the scalar fields lying on the visible brane. The curly lines denote the gravitinos and the solid lines stand for spinor fields of the radion multiplet. The blobs on top of each diagram are fermionic mass insertions at the hidden brane.

Fig[1]
- General structure of the loops involved

\[ \sim \int \frac{d^4 p}{(2\pi)^4} T_V \left[ V_G(p,0,\pi R)V_1G(p,\pi R,\pi R)V_2G(p,\pi R,0) \right] \]

All space – time indices are suppressed

\( V_1, V_{1,2} \) vertices on the visible and hidden brane respectively.

\( G(p,z,z') \): denote propagations between \( z \) and \( z' \) points for the gravitino and fermion fields carrying loop momentum \( p \).

Each diagram contributes to the scalar masses:

\[ \Delta m^2_\phi = c_{(i)} \frac{\kappa^2}{27} m^{3/2} \frac{\zeta(3)}{\pi^5 R^3} \]
\[ m_{\phi}^2 = \frac{\kappa_5^2 \zeta(3)}{16\pi^5 R^3} m_{3/2}^2 = \frac{\zeta(3)m_{3/2}^2}{\pi^3 R^2 M_{Planck}^2} \]

\[ (m_{3/2} = \kappa_5^2 |c| / \pi R) \]

NOTICE:
- In our approach \( \Delta m_{\phi}^2 \) turns to be positive. That is related to the presence of the spinor field \( \psi_5 \) which cannot be gauged away by a transformation from the whole Langrangian as an ordinary goldstino in the “unitary” gauge.

[Bagger-Belyaev-Benakli-Moura]

Diagram [ f ] involving only the gravitino has negative contribution in our case as well.

[Strumia-Scrucca-Rattazzi,Ghergetta-Riotto]
TRILINEAR SOFT SCALAR COUPLINGS

The relevant Langrangian for this computation

\[-W(\phi) \left[ \bar{\psi}_m^T P_R C \left( \gamma^{mn} - \frac{1}{3} \gamma^m \gamma^n \right) P_R \bar{\psi}_n^T + \frac{3}{2} \psi^T P_L C \bar{\psi}_L \psi - \frac{i}{2} \bar{\psi}_m P_R \gamma^m P_L \psi + h.c. \right] \]

We consider a cubic superpotential

On the **VISIBLE BRANE** \[ \rightarrow W(\phi) = \frac{\lambda}{6} \phi^3 \]

The graphs one needs to calculate are shown in the following figure

\[ c_{(a)} = 1/12 \]

\[ c_{(b)} = 1/48 \]

\[ c_{(c)} = 1/12 \]

fig.[2]
The contribution of each diagram in Fig[2] is of the form:

\[ c_{(i)} m_{3/2} \kappa_5^2 \frac{\zeta(3)}{\pi^5 R^3} W(\varphi) + h \cdot c \]

Which give a total correction to the cubic potential due to SUSY breaking occurring on the hidden brane.

\[ \frac{3}{16} m_{3/2} \kappa_5^2 \frac{\zeta(3)}{\pi^5 R^3} W(\varphi) + h \cdot c \]

Therefore the induced trilinear softscalar coupling is

\[ A = 3 m_{\varphi}^2 / m_{3/2} \]
Discussion - Highlights

- We considered in the context of $D = 5, N = 2$ SUGRA compactified on $S^1/Z_2$, the $N = 1$ supersymmetric couplings of matter localized on one of the branes.
- Inclusion of radion multiplet coupling up to $O(\kappa_5^2)$ working directly in the on-shell scheme.
- Proper treatment of the radion multiplet shows that the transmission of SUSY breaking results to $m_\phi^2 > 0$.
- A universal trilinear soft scalar coupling is induced due to SUSY breaking: $A = 3m_\phi^2/m_{3/2}$.

Future Work:

- Extend results to more realistic models (for example inclusion of gauge interactions).
- Derive $\mathcal{L}$ to all orders in $\kappa_5$. 