A few comparisons between string theory and heavy-ion physics

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Based largely on work with J. Friess, G. Michalogior-gakis, S. Pufu, and A. Yarom

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1. Why $\mathcal{N} = 4$ super-Yang-Mills?

Why compare $\mathcal{N} = 4$ super-Yang-Mills theory to the quark-gluon plasma?

- Results to date suggest that there’s a chance it might work “fairly well.”
- $S_{\text{strong}} = \frac{3}{4}S_{\text{weak}}$ and $\eta/s = 1/4\pi$ are an encouraging start.
- It’s the simplest construction, so there’s less room for theoretical fudge.
- $\mathcal{N} = 4$ computations should provide a baseline of comparison for other gauge-string constructions.

How good agreement should we hope for?

- Can’t be perfect because $\mathcal{N} = 4$ isn’t QCD! When agreement is good, it could be for wrong reasons.
- $\mathcal{N} = 4$ has conformal symmetry and about $3 \times$ as many degrees of freedom as QCD.
- Confinement and $\chi_{\text{SB}}$ are missing both in $\mathcal{N} = 4$ and in the QGP.
- Within a factor of 2 for a variety of observables? Better for clever ratios?
What is $\mathcal{N} = 4$?

Field content:

\[ A_\mu, \psi_1, \psi_2, \psi_3, \psi_4, X_1, X_2, X_3, X_4, X_5, X_6 \]

$\psi_i$ and $X_I$ are massless adjoints.

Lagrangian:

\[ \mathcal{L} = -\frac{1}{2g_{YM}^2} \text{tr} F_{\mu\nu}^2 + \text{(superpartners)} \]

Key property: $\beta(g_{YM}) = 0$: conformal.

D-brane representation:

At finite temperature, a horizon forms, with $S \approx A/4G_N$ and $T = T_{\text{Hawking}}$:

AdS/CFT provides translation between gravitational and gauge theory descriptions.
Most AdS/CFT calculations rely on expansions in large $N = 3$ (number of colors) and ’t Hooft coupling $\lambda = g^2_{YM} N = 3g_{YM}^2 = 12\pi \alpha_{SYM} \sim 12\pi \alpha_s \sim 6\pi \approx 19$. 

$1/N$ corrections $\rightarrow$ quantum gravity corrections in $AdS_5$. 

$1/\lambda$ corrections $\rightarrow$ classical string theory corrections to GR. 

Taking figure 1 as a guide, strong coupling methods might fail for $\lambda \approx 8$; perturbative methods might fail for $\lambda \approx 1$; and HTL methods might fail for $\lambda \gtrsim 4$. 

![Figure 1: HTL (green) calculations of entropy in QCD and $\mathcal{N} = 4$ [Blaizot et al. 2006]. The Padé curve agrees with best estimates at large and small ’t Hooft coupling $\lambda = g^2_{YM} N$.](image-url)
2. **Heavy ion collisions at RHIC**

- RHIC collides gold nuclei moving with $\gamma \approx 100$, so $E_{\text{tot}} \approx 39$ TeV.
- About 7500 particles come out, $\gg 197$ nucleons per gold nucleus.
- The nuclear matter is probably thermal from about $0.6 \text{ fm/c}$ to about $6 \text{ fm/c}$. (i.e. until $t = 10^{-23}$ s).
- A typical temperature is 250 MeV, above the confinement and $\chi_{SB}$ transition at $T_c \approx 170$ MeV.
- The QGP is probably strongly coupled, as evidenced by $\eta/s \ll 1$. 
**Introduction to jet-quenching**

A standard single-particle measure of energy loss in the QGP is the “nuclear modification factor,”

\[ R_{AA} \equiv \frac{dN(\text{gold-gold})/dp_Td\eta}{\langle N_{\text{binary}} \rangle dN(\text{proton-proton})/dp_Td\eta}. \]  

(1)

\( \eta \) is the “pseudo-rapidity,” \( \tanh \eta \equiv \cos \theta \).

Small \( R_{AA} \) means that hard partons dump energy very quickly as they plow through the QGP.

\( \Delta E = 10 \ \text{GeV} \) can occur in about \( 1 \ \text{fm} \).

Nuclear modification factor \( R_{AA} \) for photons and hadrons in 0 to 10% central gold-gold collisions. From [Muller and Nagle 2006].
3. Forces on a heavy quark

A heavy external quark moving at speed $v$ through thermal plasma of $\mathcal{N} = 4$ experiences a drag force [Herzog et al. 2006; Gubser 2006a]

$$\frac{dp}{dt} = -\frac{\pi \sqrt{\lambda}}{2} T^2 \frac{v}{\sqrt{1 - v^2}} \approx -\frac{\pi \sqrt{\lambda}}{2} T^2 \frac{p}{m}. \quad (2)$$

(2) arises in a simple way: a fundamental string trails out behind the quark into $AdS_5$-Schwarzschild, pulling back upon it. Scaling can be understood simply:

- $dp/dt \propto p/m$ is a surprise: Lorentz enhancement over $J \cdot A$ expectations.
A few comparisons, Gubser, Miami 2007

3 Forces on a heavy quark

• \( dp/dt \propto \sqrt{\lambda} \) because this is how string tension scales: \( L^2/\alpha' = \sqrt{\lambda} \) where \( L \) is radius of \( AdS_5 \).

Drag force is not the whole story: in a Langevin description [Casalderrey-Solana and Teaney 2006; Gubser 2006b; Casalderrey-Solana and Teaney 2007]

\[
\frac{dp}{dt} = -\eta p + F(t) \quad \eta = \frac{\pi \sqrt{\lambda} T^2}{2m} 
\]

where \( F \) is a stochastic force: if \( p \) is in the \( \hat{1} \) direction, then

\[
\langle F_1(t_1)F_1(t_2) \rangle \approx \kappa_L \delta(t_1 - t_2) , \quad \kappa_L = \pi \sqrt{\lambda} \frac{T^3}{(1 - v^2)^{5/4}} 
\]

\[
\langle F_i(t_1)F_j(t_2) \rangle \approx \kappa_T \delta_{ij} \delta(t_1 - t_2) , \quad \kappa_T = \pi \sqrt{\lambda} \frac{T^3}{\sqrt{4} \sqrt{1 - v^2}} 
\]

where \( i, j = 2, 3 \). Sometimes I prefer to quote

\[
\hat{q}_T \equiv \frac{\langle p_{\perp}^2 \rangle}{\lambda} = \frac{2\kappa_T}{v} = 2\pi \sqrt{\lambda} \frac{T^3}{v^4 \sqrt{1 - v^2}} ;
\]

however this definition of \( \hat{q}_T \) differs from \( \hat{q} \) in [Liu et al. 2006].
String theory value for $\kappa_L$ exceeds Einstein relation except near $v = 0$:

$$\kappa_L = \frac{1}{(1 - v^2)^{3/4}} 2T E \eta,$$  \hspace{1cm} (6)

hinting that Langevin description doesn’t capture all the physics.

Another such hint comes from a scaling form for force correlators:

$$\langle F_i(t_1) F_j(t_2) \rangle = \delta_{ij} G_T(t_2 - t_1)$$

$$G_T(t) = \pi T^3 \frac{\sqrt{\lambda}}{\sqrt[4]{1 - v^2}} g_T(\ell)$$

where $\ell \equiv \sqrt[4]{1 - v^2} \pi T t$  \hspace{1cm} (7)

The take-away message: correlation time in $\vec{F}(t)$ diverges as $1/\sqrt[4]{1 - v^2}$. 

The force correlator as a function of dimensionless time. The red curve is $g_T(\ell)$; other curves are estimates and corrections. From [Gubser 2006b].
The way one gets at $\vec{F}$ in string theory is by noting that the trailing string worldsheet has a causal horizon at $y = y_v$. Consider a point on the string at fixed height $y$ in $AdS_5$-Schwarzschild:

$$ds_5^2 = \frac{L^2 \pi^2 T^2}{y^2} \left[ -(1 - y^4) dt^2 + dx^2 + \frac{1}{\pi^2 T^2} \frac{dy^2}{1 - y^4} \right]. \quad (8)$$

- $d\tau^2 > 0$ if $y > y_v \equiv \sqrt{1 - v^2}$: “outside” the worldsheet black hole.
- $d\tau^2 < 0$ if $y < y_v$: “inside” the worldsheet black hole.

Something roughly like Hawking radiation must emanate from the worldsheet horizon, leading to stochastic $\vec{F}(t)$. Actual computations directly access $\langle F_i(t_1) F_j(t_2) \rangle$. 
Naive comparison with data

\[ \eta = \frac{\pi \sqrt{\lambda T^2}}{2m} \Rightarrow \]

\[ 2\pi T D = \frac{2\pi T^2}{m\eta} = \frac{4}{\sqrt{\lambda}} \]

which is too small if \( \lambda \sim 19 \), corresponding to \( \alpha_s = 1/2 \).

In other words,

\[ t_{\text{charm}} = \frac{1}{\eta} \sim 0.6 \text{ fm} \]

is too small for \( R_{AA} \). But see section 6...

Figure 4: \( R_{AA} \) and \( v_2 \) for heavy quarks. \( p_T \) is for a non-photonic electron. From [Adare et al. 2006].
It was recently suggested [Horowitz and Gyulassy 2007] that a distinctive difference between pQCD and AdS/CFT predictions from RHIC to LHC energies comes from

\[ R_{AA}^{cb} \equiv \frac{R_{AA}^{b}}{R_{AA}^{c}} \sim \left\{ \frac{t_{\text{bottom}}}{t_{\text{charm}}} \approx \frac{m_{\text{charm}}}{m_{\text{bottom}}} \quad \text{for AdS/CFT} \right\} \]

\[ 1 - \frac{p_{cb}}{p_T} \quad \text{for pQCD}, \quad p_{cb} \propto \hat{q} L^2 \]  

Figure 5: pQCD predictions for \( R_{AA}^{cb} \) separate cleanly from AdS/CFT because assumptions about initial conditions cancel out. But beware uncertainty on the limits of validity of AdS/CFT.
4. Diffusion wakes and sonic booms

“Pull-back” from the string is a five-dimensional metaphor for momentum transfer from the quark to the thermal bath. Can we be more precise? Yes: [Friess et al. 2006a; Yarom 2007; Gubser et al. 2007b; Chesler and Yaffe 2007; Gubser et al. 2007a]....

- Tree-level graviton propagation from string to boundary tells us $\langle T_{mn} \rangle$ in the gauge theory.

Figure 6: In blue: the trailing string of an external quark. The dashed line shows classical propagation of a graviton from the string to the boundary, where its behavior can be translated into the stress-energy tensor $\langle T_{mn} \rangle$ of the boundary gauge theory [Friess et al. 2006a].
• Easiest to solve linearized Einstein’s equations via a co-moving ansatz:

\[ h_{mn}(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i k_1 (x_1 - vt) + i k_2 x_2 + i k_3 x_3} h_{mn}(\vec{k}). \]  

(12)

• Prefer to render all quantities dimensionless with factors of \( \pi T \), e.g.

\[ \vec{K} = \frac{\vec{k}}{\pi T}, \quad \vec{X} = \pi T \vec{x}. \]  

(13)

• Angle \( \theta \) between \( \vec{v} \) and \( \vec{k} \) is roughly \( \pi - \Delta \phi \), where \( \Delta \phi \) is opening angle in dihadron correlators.

\[ K_1 = K \cos \theta, \quad K_p = K \sin \theta, \quad X_1 = X \cos \theta, \quad X_p = X \sin \theta. \]  

(14)

• Rescale energy density and subtract off thermal bath as well as vacuum field of the quark:

\[ E(\vec{X}) \equiv \frac{\sqrt{1 - v^2}}{(\pi T)^4 \sqrt{\lambda}} \left\langle T^{00}(0, \vec{x}) - T_{\text{bath}}^{00} - T_{\text{Coulomb}}^{00}(0, \vec{x}) \right\rangle \]  

(15)

and likewise define dimensionless Poynting vector \( \vec{S} \) in terms of \( T^{0i} \).
Figure 7: For $T \approx 318$ MeV, $|\vec{X}| = 1$ is a distance $0.2$ fm from the quark. From [Gubser et al. 2007a].
Some **good** news, some **bad**:

- Mach angle is too small to fit data well: \( c_s = 1/\sqrt{3} \), so \( \theta_M \approx 55^\circ \), wanted 70°.
- We get \( E \) and \( \vec{S} \) at *all* scales, from 0.01 fm to 30 fm, from one (big) computation with no free parameters except \( v \).
- There’s no RG flow, so comparisons with QCD below 0.1 fm are more fraught than usual.
- Medium is infinite and static.
- There is a strong Mach cone. Match to linearized hydro with \( \eta/s = 1/4\pi \) is great for \(|\vec{X}| \gtrsim 2\) fm.
- Energy is lost to sound modes almost *twice* as fast as the *total* power loss.
- Diffusion wake is also strong.
- **Remember**, this is for a heavy quark as the away-side parton!

Accounting depends on energy flowing *in toward the quark* in diffusion wake:

\[
\text{(sound)} : \text{(diffusion)} : \text{(total)} = 1 + v^2 : -1 : v^2
\]

The scaling (16) is fairly universal [Gubser and Yarom 2007].
Comparison with data requires hadronization...

...and earlier hydro studies [Chaudhuri and Heinz 2006; Casalderrey-Solana et al. 0200] suggest that it’s uphill work, even if diffusion wake is suppressed by hand.

Counter-clockwise from lower-left: PHENIX [Jia 2007], STAR [Adams et al. 2005], hydro w/o diffusion wake \( \frac{dE}{dx} = 10T^2 \) and various \( p_T \) [Casalderrey-Solana et al. 0200].
5. Expanding plasmas and thermalization time

Finding gravity dual of a RHIC collision is hard work, but a special type of spherically symmetric radial flow is easy: the “conformal soliton,”

\[
\rho(t, r) = \rho_{\text{peak}} \frac{3L^4 + 6r^2L^2 + 3r^4 + 6t^2L^2 + 10r^2t^2 + 3t^4}{3(L^4 + 2r^2L^2 + r^4 + 2t^2L^2 - 2r^2t^2 + t^4)^3},
\]

which is conformal to a static plasma on \( S^3 \). (Obvious going the other way.)

No shear, no entropy production. But if perturbed, it re-equilibrates. How fast?

There’s no scale in a CFT; must therefore set \( T_{\text{peak}} \sim 300 \text{ MeV/fm}^3 \) and \( L \sim 7 \text{ fm} \) to “compare” with a RHIC collision.
The \( e \)-folding time for the slowest non-hydrodynamical mode is

\[
\tau_{e\text{-fold}} \approx 1/8.6T_{\text{peak}}.
\]  (18)

Extrapolating to RHIC flows, where initial state is highly anisotropic, a rough estimate of thermalization / isotropization time is

\[
\tau_{\text{therm}} \sim 4\tau_{e\text{-fold}} \approx 0.3 \text{ fm}/c.
\]  (19)

One arrives at (18) from studying linearized perturbations of the finite-mass black hole dual to the conformal soliton [Friess et al. 2006b].

Perhaps, with sufficient effort, finite-mass black holes in \( AdS_5 \) could be collided: dual to collisions of boosted blobs of QGP.
6. Comparing $\mathcal{N} = 4$ to the QGP

Consider two schemes for making comparisons:

\begin{align*}
\text{obvious:} & \quad T_{SYM} = T_{QCD} = 250 \text{ MeV} \quad g_{YM}^2 N = 12\pi \alpha_s = 6\pi \approx 19 \\
\text{alternative:} & \quad 3^{1/4}T_{SYM} = T_{QCD} = 250 \text{ MeV} \quad g_{YM}^2 N = 5.5. \quad (20)
\end{align*}

“Obvious” doesn’t need much justification: $\alpha_s = 1/2$ is a widespread rule-of-thumb estimate in the specified temperature range, and $g_{YM}^2 N = 12\pi \alpha_s$ corresponds to $g_{YM} = g_s$.

I arrived at “alternative” by comparing string predictions for $q\bar{q}$ to lattice computations. Lattice people define an effective coupling:

$$
\alpha_{qq}(r, T) \equiv \frac{3}{4} r^2 \frac{\partial F_{qq}}{\partial r}. \quad (21)
$$

Analogous quantity in string theory receives contributions from two configurations, shown in figure 8.

- Only the U-shape is fully understood, and I include only it in later plots (figure 9).
Figure 8: Two string configurations contributing to $\alpha_{\text{SYM}}(r, T)$.

- There are attractive interactions between the anti-parallel strings that dominate the $\tilde{r} > \tilde{r}_*$ regime, and recent work [Bak et al. 2007] has endeavored to quantify them and compare to lattice QCD.

Simplest approximation to U-curve contribution is zero temperature result:

$$\alpha_{\text{SYM}}(T = 0) \equiv \frac{3}{4} r^2 \frac{\partial V_{q\bar{q}}}{\partial r} = \sqrt{g_{YM}^2 N} \frac{3\pi^2}{\Gamma(1/4)^4}.$$ (22)

$T \neq 0$ results in a bit of Debye screening.

- To fix $g_{YM}^2 N \approx 5.5$, compare to lattice at largest $r$ where U-shape dominates. Overlap of lattice and SYM is a bit better when one compares at fixed energy density rather than fixed temperature.
Figure 9: Static quark force for $\mathcal{N} = 4$ SYM (yellow band) versus $N_f = 2$ lattice results (colored dots from [Kaczmarek and Zantow 2005]. If $T_c = 170$ MeV, then $T = 209$ MeV (red dots), 233 MeV (green dots), 255 MeV (blue dots). Dark center line in yellow is $g_{YM}^2 N = 5.5$. Dashed grey is from Cornell potential. The choice $T_{SYM} = 190$ MeV amounts to fixed energy density comparison with QCD. From [Gubser 2006c].

- Makes sense: more matter, faster thermal screening.
- $\epsilon_{SYM} = \epsilon_{QCD}$ means $T_{SYM} \approx T_{QCD}/3^{1/4}$.
- Match between SYM and lattice here is conspicuously imperfect, but I wanted some comparison where leading-order result on SYM side involves $g_{YM}^2 N$.
- An alternative perspective can be found in [Sin and Zahed 2007].
### Table 1: A few comparisons between $\mathcal{N} = 4$ SYM and the QGP.

QGP numbers are representative ranges. $T = 250$ MeV unless otherwise noted. $T_c = 170$ MeV, $m_c = 1.4$ GeV assumed.

<table>
<thead>
<tr>
<th>quantity</th>
<th>formula</th>
<th>obvious</th>
<th>alternative</th>
<th>QGP</th>
<th>comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s/s_{\text{free}}$ [1,2]</td>
<td>$\frac{3}{4} + \frac{45\zeta(3)}{32\lambda^{3/2}}$</td>
<td>0.77</td>
<td>0.88</td>
<td>0.6 – 0.9</td>
<td>lattice e.g. [3] HTL [4,5]</td>
</tr>
<tr>
<td>$\eta/s$ [6,7]</td>
<td>$\frac{1}{4\pi} + \frac{135\zeta(3)}{32\pi\lambda^{3/2}}$</td>
<td>0.10</td>
<td>0.2</td>
<td>0 – 0.3</td>
<td></td>
</tr>
<tr>
<td>$\tau_{\text{therm}}$ [8]</td>
<td>$\frac{1}{2.2T_{\text{peak}}}$</td>
<td>0.3 fm</td>
<td>0.4 fm</td>
<td>0.6 – 1.0 fm</td>
<td>Also [9,10] $T_{\text{peak}}=300$ MeV</td>
</tr>
<tr>
<td>$t_{\text{charm}}$ [11,12,13]</td>
<td>$\frac{2m_c}{\pi T^2 \sqrt{\lambda}}$</td>
<td>0.65 fm</td>
<td>2.1 fm</td>
<td>3 – 6 fm</td>
<td>[14,15,16] hadronization $b$-tagging</td>
</tr>
<tr>
<td>$\hat{q}$ [17]</td>
<td>$\frac{\pi^{3/2} \Gamma(3/4) \sqrt{\lambda T^3}}{\Gamma(5/4)} \frac{\sqrt{\lambda \gamma T^3}}{\nu}$</td>
<td>2.6 GeV</td>
<td>0.61 GeV</td>
<td>1 – 15 GeV</td>
<td>[20] for QGP $\hat{q}_T$ for charm @ $E=10$ GeV</td>
</tr>
<tr>
<td>$\hat{q}_T$ [18,19]</td>
<td>$\frac{\pi^{3/2} \Gamma(3/4) \sqrt{\lambda T^3}}{\Gamma(5/4)} \frac{\sqrt{\lambda \gamma T^3}}{\nu}$</td>
<td>5.8 GeV</td>
<td>1.4 GeV</td>
<td>1 – 15 GeV</td>
<td></td>
</tr>
<tr>
<td>$m_D$ [21]</td>
<td>$(10.6 - \frac{6.7}{\lambda^{3/2}}) T$</td>
<td>3.6 GeV</td>
<td>2.6 GeV</td>
<td>$\sim 1.9$ GeV</td>
<td>lattice [22] $T=340$ MeV 6.7 conjectural</td>
</tr>
</tbody>
</table>
References from the comparison table

[8] Friess, Gubser, Michalogiorgakis, and Pufu 2006b
[9] Lublinsky and Shuryak 2007
[12] Casalderrey-Solana and Teaney 2006
[14] van Hees and Rapp 2005
[16] Adare et al. 2006
[17] Liu, Rajagopal, and Wiedemann 2006
[18] Gubser 2006b
[21] Bak, Karch, and Yaffe 2007
7. Outlook

There are a handful of interesting comparisons between string theory and the QGP produced at RHIC. Maybe more will come.

- Quantitative comparisons aren’t all spot-on, but possibly within a factor of 2 if one compares at fixed energy density.

- It would be helpful if theorists could:
  1. Find a gravity dual of QCD that works at finite temperature and has minimal theoretical fudge.
  2. Get better control of $\alpha'$ corrections.
  3. Bridge better to phenomenological studies, e.g. by including hadronization.

- It would be helpful if experimentalists could:
  1. Constrain $\Delta \phi \approx \pi$ region of di-hadron correlators in a way I can compare better between STAR and PHENIX. Is the diffusion wake ruled out?
  2. Tag $b$’s and $c$’s. How about 2pt function of HQ and hard associated hadron?
  3. Keep running: some heavy-quark measurements look luminosity-limited, e.g. centrality-specific $v_2$ for charm.
More thoughts for the future

- How good is the match of AdS/CFT to hydrodynamics beyond linearized approximation? See e.g. the figure below from [Noronha et al. 2007]. Can AdS/CFT elucidate turbulence and/or high-gradient fluid flows?
- When is AdS/CFT going beyond hydro? Is it more to RHIC physics than a minimal interpolation between CFT and low-viscosity hydro?

- Thermalization time is important to heavy-ion physics, and existing treatments (including the one I discussed) are not very satisfactory.

- Need to consider stochastic dynamics in violation of Einstein relation, or with a composite quark-string system. Inclusion of fluctuations in current AdS/CFT treatments of hard probes seems feeble.
• Diffusion wake or no diffusion wake? Is this affected by fluctuations? Turbulence? Expanding medium? Is the wake related to near side ridge and/or directed baryon excess, as Jacak has speculated? Figure from [Wong 2007].

![Graph](image)

• **The good news**: AdS/CFT has made string theory a player in the phenomenology of a modern, data-rich, experimental field.

• **However**: Playing and winning are not necessarily the same thing.
References


