Topology from Cosmology

Per Berglund
University of New Hampshire

based on arXiv:0712:1815 with
Balasubramanian, Jimenez, Simon, Verde

Miami 2007 2007/12/15
General Idea and Outline

Study how cosmological observables depend on the microscopic parameters. In particular, use cosmological observations to constrain the topology of the extra dimensions

Outline

• Background--N=1 supergravity and Large Volume Scenario
• Slow-roll inflation with multi-fields
• Cosmological observables
• Dependence of observables on the microscopics
• Conclusion
Background (I)

- We consider N=1 supergravity action from type IIB orientifold compactification on Calabi-Yau manifold

\[ S_{N=1} = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - g_{i\bar{j}} D_\mu \phi^i D^\mu \bar{\phi}^j - V(\phi_i, \bar{\phi}_i) \right] \]

\[ V(\phi_i, \bar{\phi}_i) = e^{\mathcal{K}/M_P^2} \left( g^{i\bar{j}} D_i \hat{W} D_{\bar{j}} \hat{W} - \frac{3}{M_P^2} \hat{W} \hat{W} \right) + V_{\text{uplift}} \]

\[ D_i \hat{W} = \partial_i \hat{W} + \hat{W} \partial_i \mathcal{K} \]

\[ g_{i\bar{j}} = \partial_i \partial_{\bar{j}} \mathcal{K} \]

\[ \mathcal{K} = -2M_P^2 \ln \left( \mathcal{V} + \frac{\xi}{2} \right) - \ln \left( \frac{2}{g_s} \right) + \mathcal{K}_0 \]

\[ \mathcal{V} = \mu \tau_1^{3/2} - \lambda \tau_2^{3/2} = \frac{1}{x} \left( 1 - \lambda x \tau_2^{3/2} \right), \quad x^{-1} \equiv \mu \tau_1^{3/2} \]

\[ \hat{W} = \frac{g_s^2 M_P^3}{\sqrt{4\pi}} \left( W_0 + \sum_i A_i e^{-a_i T_i} \right) \]
We consider the so-called Large Volume Scenario which allows us to perform a controlled expansion of the potential in powers of the inverse volume:

\[ \beta_1 \ll 1, \quad \beta_2 \ll 1, \quad \frac{A_2 e^{-y}}{|W_0|} \ll 1 \quad \beta_1 \equiv x \frac{\xi}{2}, \quad \beta_2 \equiv \lambda x \tau_2^{3/2} \quad y \equiv a_2 \tau_2 \]

\[
V = M_P^4 \hat{\gamma} x_0^2 \left( \frac{x^2}{x_0^2} + 3z_3 \frac{x^3}{x_0^3} + 4z_2 \frac{y}{\sqrt{y_0}} \frac{x}{x_0} e^{2(y_0-y)} - 2z_1 \frac{x^2}{x_0^2} \frac{y}{y_0} e^{y_0-y} \right)
\]

\[
z_1 = \frac{g_s^4 |W_0|^2}{4\pi \hat{\gamma}} \frac{A e^{-y_0}}{|W_0|} y_0, \quad z_2 = \frac{g_s^4 |W_0|^2}{4\pi \hat{\gamma}} \frac{y_0^2}{3\beta_0} \left( \frac{A e^{-y_0}}{|W_0|} \right)^2, \quad z_3 = \frac{g_s^4 |W_0|^2}{4\pi \hat{\gamma}} \frac{1}{8} \xi x
\]

\[
\beta_0 = \beta_2 |x=x_0, y=y_0| \quad \hat{\gamma} \equiv \gamma/M_P^4
\]

The combination of microscopic parameters \(z_i\) will be constrained by the cosmological observables.
Background (III)

• At the large volume minimum \( x^{-1} \sim e^y \quad y = O(10) \)

• We are interested in rolling in the (inverse) x-direction

• Finally, we diagonalize the metric/kinetic energy

\[
K = \frac{1}{2} \left( (\dot{q}^1)^2 + e^{\sqrt{6} q^1 / M_{pl}} (\dot{q}^2)^2 \right) = \frac{1}{2} \left( (\dot{q}^1)^2 + M_{pl}^2 \frac{4}{3} \beta_2 \left( \frac{\dot{q}^2}{q^2} \right)^2 \right)
\]

\[
q^1 = M_{pl} \sqrt{\frac{3}{2}} \log \tau_1, \quad q^2 = M_{pl} \frac{2}{\sqrt{3}} \sqrt{\frac{\lambda}{\mu}} \left( \tau_2 \tau_1^{-3} \right)^{\frac{3}{4}}
\]
Slow-roll (I)

- The equations of motions are given by, where

\[ H^2 = \frac{1}{3 \, M_{pl}^2} (K + V) \quad \frac{d^2 q^i}{dt^2} + 3H \dot{q}^i = -G^{ij}_{\cdot \cdot} \frac{\partial V}{\partial q^j} \]

\[ \frac{d^2 q^i}{dt^2} \equiv \ddot{q}^i + \Gamma^i_{jk} \dot{q}^j \dot{q}^k \quad \frac{d}{dt} = \dot{q}^i \nabla_i \quad \Gamma^i_{jk} = \frac{1}{2} G^{il}_{\cdot \cdot} \left( \frac{\partial G_{lk}}{\partial q^j} + \frac{\partial G_{lj}}{\partial q^k} - \frac{\partial G_{jk}}{\partial q^l} \right) \]

- Due to the large number of Kahler moduli we need to study the multi-field version of slow-roll

\( (1) \quad K \ll V \quad \Rightarrow \quad H^2 \approx \frac{V}{3 \, M_{pl}^2} \)

\[ \hat{\epsilon} \equiv \frac{K}{M_{pl}^2 \, H^2} = \frac{\dot{q}^i \dot{q}^i}{2 \, M_{pl}^2 \, H^2} \ll 1 \]

\( (2) \quad \left| \frac{d^2 q^i}{dt^2} \right| \ll \left| 3H \dot{q}^i \right| \quad \Rightarrow \quad 3H \dot{q}^i \approx -G^{ij}_{\cdot \cdot} \frac{\partial V}{\partial q^j} \quad \left| \frac{K}{V} - \frac{M_{pl}^2 \, \dot{q}^k V_{;k}^i}{3V \, \dot{q}^i} \right| \ll 1 \]

which gives us

\[ \frac{M_{pl}^2 \, \dot{q}^k V_{;k}^i}{3V \, \dot{q}^i} \ll 1 \quad \forall i \]

\[ \hat{\eta} \equiv \frac{M_{pl}^2}{6H^2 \, K} \dot{q}^i \dot{q}^j V_{;ij} \ll 1 \]

Hence a total of \( n+1 \) slow-roll conditions
Slow-roll (II)

• We now apply this to the Large Volume Scenario

\[
\dot{\epsilon} \approx \frac{M_{pl}^2}{2} \left( \frac{V_1}{V} \right)^2 \quad \quad \hat{\eta} \approx M_{pl}^2 \frac{V_{11} V_{22} - (V_{12})^2}{V V_{22}}
\]

where we for the second slow-roll constraint have used the following relations

\[
\left| V_{;11} + \frac{q^2}{q^1} V_{;12} \right| \ll \frac{V}{M_{pl}^2} \quad \quad \left| G^{22} V_{;22} + \frac{q^1}{q^2} G^{22} V_{;21} \right| \ll \frac{V}{M_{pl}^2}
\]

and with \( q^2 \) the heavy field we solve by setting the second relation to zero

\[
\frac{\dot{q}^2}{\dot{q}^1} \approx -\frac{V_{;12}}{V_{;22}}
\]

or using the slow-roll equations of motion

\[
\frac{V_2}{V_1} \approx -\frac{V_{;12}}{G^{22} V_{;22}}
\]
Observables (I)

• Adiabatic power spectrum

\[ <Q^*_\sigma k Q_\sigma k'> = \frac{2\pi^2}{k^3} P(k) \delta(k - k') \]

\[ P \approx \left( \frac{H^2}{2\pi \dot{\sigma}} \right)^2 = \frac{H^4}{2(2\pi)^2 K} \approx \frac{2}{3\pi^2} \frac{V}{M_{pl}^4} \frac{1}{16 \dot{\epsilon}} \]

which in the Large Volume Scenario becomes

\[ P \approx \frac{1}{12\pi^2} \frac{V^3}{M_{pl}^6 V_1^2} \]

• The spectral index

\[ n_s - 1 = \frac{d \log P}{d \log k} \approx H^{-1} \frac{\dot{P}}{P} = -6 \dot{\epsilon} + 2 \dot{\eta} \]

which in the Large Volume Scenario becomes

\[ n_s - 1 \approx M_{pl}^2 \left( -3 \left( \frac{V_1}{V} \right)^2 + 2 \frac{V_{11}}{V} \left( 1 - \frac{(V_{12})^2}{V_{11}V_{22}} \right) \right) \]
Observables (II)

- The running of the spectral index

\[
\alpha_s = \frac{dn_s}{d\log k} \approx H^{-1} \frac{dn_s}{dt} \approx -24(\dot{\epsilon})^2 + 16\dot{\epsilon} \dot{\eta} - 2\hat{\psi} + 4 \left( (\hat{\eta})^2 - \left( \frac{\eta^2}{2} \right) \right)
\]

\[
\left( \frac{\eta^2}{2} \right) = \frac{1}{18H^4K} \dot{q}^i \dot{q}^k V_{ik} V_{ij}, \quad \hat{\psi} = -\frac{1}{6H^3K} \dot{q}^i \dot{q}^j \dot{q}^k \nabla_k (V_{ij})
\]

which in the Large Volume Scenario becomes

\[
\alpha_s \approx -24(\dot{\epsilon})^2 + 16\dot{\epsilon} \dot{\eta} - 2\hat{\psi}
\]

where

\[
\hat{\psi} \approx M_{pl}^4 \frac{V_1 V_{111}}{V^2} \left( 1 - 3 \frac{V_{112}}{V_{111}} \frac{V_{12}}{V_{22}} + 3 \frac{V_{122}}{V_{111}} \left( \frac{V_{12}}{V_{22}} \right)^2 + \frac{V_{222}}{V_{111}} \left( \frac{V_{12}}{V_{22}} \right)^3 \right)
\]

since

\[
\left( \frac{\eta^2}{2} \right) \approx 0(\beta_0)
\]

- The tensor to scalar ratio

\[
r = 16 \dot{\epsilon} \approx 8M_{pl}^2 \left( \frac{V_1}{V} \right)^2
\]
Observables (III)

• We can now solve for the microscopic parameters $z_1, z_2, z_3$ in terms of $r$ and $n_s$

• Prediction for the running in the limit of small $r$ and large $y_0$

$$\alpha_s \approx 23 s \sqrt{r} \quad n_s \sim 0.95 \quad s = \text{sign}(V_1)$$

• Because of the experimental bound $|\alpha_s| < 0.1$ it requires $r < 10^{-4}$ so any detection of gravitational waves in the near future would rule out this model

• Relation between power spectrum, the tensor to scalar ratio and the volume of Calabi-Yau since

$$\frac{\xi x_0}{2} \sim \frac{8\pi}{5} \frac{\hat{\gamma} y_0}{g_s^4 |W_0|^2}$$

$$\mathcal{P} \approx \frac{2}{3\pi^2} \frac{\hat{\gamma} x_0^2}{r} (1 + 3z_3 + 4z_2 - 2z_1) \approx 7.5 \times 10^{-3} \frac{\hat{\gamma} x_0^2}{r}$$
Observable Dependence on Microscopics

- Interesting to study how sensitive the observables are to changes in the topology of Calabi-Yau manifold, i.e. CY A \rightarrow CY B

- The discrete nature of the change in the Euler number of the Calabi-Yau means that there’s a minimum amount of change that takes place \( |\delta \chi| = 2 \) which gives \( \delta \xi / \xi \geq 10^{-3} \)

- It is possible to compensate for the change in the Euler number by a fine-tuned change in the flux superpotential to preserve the slow-roll condition on the relationship between the velocities of the two fields. \( \frac{\delta |W_0|}{|W_0|} \sim -\frac{27\beta_2}{8y_0} \frac{\delta \xi}{\xi} \)

- However, in order to stay within the observable bounds on the adiabatic power spectrum, the tensor to scalar ratio, and the spectral index, one in addition has to change the initial conditions as well

\[
\frac{\delta \mathcal{P}}{\mathcal{P}} = 3 \frac{\delta V}{V} - 2 \frac{\delta V_1}{V_1} \ll 1 \\
\frac{\delta r}{r} = 2 \frac{\delta V_1}{V_1} - 2 \frac{\delta V}{V} \\
\delta (n_s - 1) \approx -6\delta \hat{e} + 2\delta \hat{n} \ll 1
\]
• We have additional observables, such as the running of the spectral index, the number of e-folds etc that will undergo similar large changes. However, we do not have any independent freely tunable parameters to our disposable.

• In a general setting there will be a large number of Kahler moduli, $n$. Although there are now $n$ initial conditions, we also have $n+1$ slow-roll conditions (before it was 2 and 2+1).

• There are additional microscopic parameters,
  
  • $A_i$ and $a_i$ are dynamically determined by complex structure moduli which are fixed by the flux superpotential and the type of enhanced gauge symmetry, respectively.
  
  • The intersection numbers $k_{ijk}$ are specified by the choice of Calabi-Yau manifold

• Thus, we’d expect it to be hard to fit the microscopic parameters to the cosmological observables or said differently, if models can be found that do fit the data, the topology of these models will be constrained
Conclusions

• Studied multi-field Kahler inflation in Large Volume Scenario
• Slow-roll conditions tightly constrain the system
• Observables depend very sensitively on the microscopic parameters
• Assuming that the 1-loop determinant, $A_i$, and the corresponding type, $a_i$, of the non-perturbative corrections to the superpotential $W$ are not independent of the flux superpotential $W_0$ the cosmological observables constrain the topology of the extra dimensional manifold